Ultra Scientist Vol. 28(1)A, 33-42 (2016). ISSN 2231-3478(Print) ISSN 2319-8052 (Online)

A Heuristic Approach to Establish an Algebraic Structure on Multi Star Granular Nano Topology

[1]M. LELLIS THIVAGAR and [2]S.P.R. PRIYALATHA

[1,2]School of Mathematics, Madurai Kamaraj University
Madurai-625021, Tamilnadu (India)
Email:[1]mlthivagar@yahoo.co.in,[2]spr.priyalatha@gmail.com

(Acceptance Date 26th February, 2016)

Abstract

In this paper we introduce a new concept in nano topology called as “Multi Star Granular Nano Topology” (MSGNT) to study multi granulation. The algebraic structure on basic set theoretic operations and De-Morgan’s law are studied in MS-GNT. Further, we show that multi granulation is better than single granulation from a decision table in nano accuracy and nano degree of dependency following heuristic approach.

Key words: Multi star granular nano topology, Multi star lower approximation, Multi star upper approximation, Multi star boundary region, Nano accuracy measure.

I. Introduction

Lellis Thivagar *et al.*[4] introduced a nano topological space with respect to a subset X of an universe which can be defined in terms of lower approximation and upper approximation and boundary region. We introduce the multi star granular nano topological space (MSGNT) has been now extensive to algebraic structure on basic set theoretic operations such as union, intersection, complement and De-Morgan’s law on the fundamental properties have also been made. In view of granular computing[2,4,5] ,nano topological space is based on a single granulation (only one indiscreniblity relation). But this concept of multi star granular nano topological model[11], where the set approximations are defined by multiple indiscreniblity relations on the universe.

In Pawlak[8] has narrated that some measures associated with rough sets can be associated with rough sets can be helpful to get an idea, how accurate is the information relation with some equivalence relation for a particular classification.

Further, the MSGNT support with propositions and examples are examined. One of the main concern of the nano accuracy measure and nano degree of dependency based on MSGNT is to analysis the complete information system. It has been available to use multi equivalence class in order to have an efficient and practical solution in MSGNT and the results are obtained and significant from the point of view that we have provide suitable examples and wherever necessary to show that the results are essentially happen.

II. *Preliminaries:*

Definition 2.1[11] : Let U be an universal set, $R \subseteq U \times U$ be an equivalence relation on U then U/R (or) R[x] we mean the family of all equivalence classes of R and [x] denotes a equivalence class containing an element $x \in U$. That is [x]={y | yRx}.

Definition 2.2[4]: Let U be a non-empty finite set of objects called the universe, R be an equivalence relation on U named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be approximation space. Let $X \subseteq U$

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is,

$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is,

$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$, where R(x) denotes the equivalence class determined by x.

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not -X with respect to R and it is denoted by denoted by $B_R(X)$. That is,

$B_R(X)=U_R(X)-L_R(X)$

Definition 2.3 [4]: Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\emptyset \in \tau_R(X)$.

(ii) The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U called as the nano topology on U with respect to X. We call $\{U, \tau_R(X)\}$ as the nano topological space.

III. *Essential Operations in MSGNT:*

In this section, we found the impression of union, intersection, complement, difference and De-Morgan's law are studied in multi star granular nano topological space (MSGNT). We have discussed in various of its properties[6,7].

Definition 3.1: Let U be the universe and R, S be any two equivalence relations on U named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another and

$\tau_{R*S}(X \cup Y) = \{U, \emptyset, L_{R*S}(X \cup Y), U_{R*S}(X \cup Y), B_{R*S}(X \cup Y)\}$ ($\tau_{R*S}(X \cap Y)$ res.,) where $L_{R*S}(X \cup Y)$, $U_{R*S}(X \cup Y)$, $B_{R*S}(X \cup Y)$, defined as follows. Let $X \cup$

YU⊆.

(i) The multi star lower approximation of X∪Y with respect to R,S is the set of all objects which can be for certain classified as X ∪ Y with respect to R and S and it is denoted by $L_{R*S}(X \cup Y)$. That is

$$L_{R*S}(\cup Y)= \bigcup_{x\in U}\{[x]: R(x)\subseteq X\cup Y$$

and $S(x)\subseteq X\cup Y\}$, where R[x] and S[x] denotes the equivalence class determined by x.

(ii) The multi star upper approximation of X ∪ Y with respect to R and S is the set of all objects which can be possibly classified as X∪Y with respect to R and S and it is denoted by $U_{R*S}(X\cup Y)$$. That is

$$U_{R*S}(X\cup Y) = \bigcup_{x\in U}\{[x]: R(x)\cap X\cup Y\neq\Phi$$

or $S(x)\cap X\cup Y\neq\Phi$ }, where R[x] and S[x] denotes the equivalence class determined by x.

(iii) The multi[1] star boundary region of X∪Y with respect to R and S is the set of all objects which can be classified neither as X∪Y nor as not X∪Y with respect to R and S and it is denoted by $B_{R*S}(X\cup Y)$$. That is

$B_{R*S}(X\cup Y)=U_{R*S}(X\cup Y)- L_{R*S}(X\cup Y)$ That is, $\tau_{R*S}(X\cup Y)$ forms a topology on U called as the multi star granular nano topology on U with respect to X∪Y. We call (U, $\tau_{R*S}(X\cup Y)$) as the multi star granular nano topological space in terms of union[3].

Example: 3.2 Let U={a,b,c,d,e}, U/R={{a,b},{c,d},{e}}and{U}/S={{a, c},{b,e},{d}} be two equivalence classes on U.

Let X ={a,b}⊆ U and Y = {a,c}⊆U, X∪Y={a,b,c}.Then $L_R(X\cup Y)$ = {a,b},$U_R(X\cup Y)$={a,b,c,d}, $B_R(X\cup Y)$ ={c,d} and also $L_S(X\cup Y)$={a,c},$U_S(X\cup Y)$={a, b,c, e}, $B_S(X\cup Y)$={b,e}. Therefore the nano topology $\tau_R(X\cup Y)$={U,∅,{a,b},{a,b,c,d},{c,d}} and $\tau_S(X\cup Y)$={U,∅,{a,c},{a,b,c,e}, {b,e}}. Now $L_{R*S}(X\cup Y)$={a},$U_{R*S}(X\cup Y)$ ={a, b, c, d,e} and $B_{R*S}(X\cup Y)$={b,c,d,e}. Hence the multi star granular nano topology $\tau_{R*S}(X\cup Y)$ ={U,∅,{a},{b,c,d,e}}.

Definition 3.3: Let U be a non empty finite set of objects called the universe and R, S be any two equivalence relations on U named as the indiscernibility relation. Elements belonging to the same equivalence classes are said to be indiscernible with one another and $\tau_{R*S}(X)^c$={U,∅, $L_{R*S}(X)^c$, $U_{R*S}(X)^c$, $B_{R*S}(X)^c$}where $L_{R*S}(X)^c$, $U_{R*S}(X)^c$, $B_{R*S}(X)^c$ defined as follows. Let $X^c\subseteq U$.

(i) The multi star lower approximation of X^c with respect to R and S is the set of all objects which can be for certain classified as X^c with respect to R and S and it is denoted by $L_{R*S}(X)^c$. That is, $L_{R*S}(X)^c= \bigcup_{x\in U}\{[x]: R(x)\subseteq X^c$

and $S(x)\subseteq X^c$ }, where R[x] and S[x] denotes the equivalence class determined by x.

(ii) The multi star upper approximation of X^c with respect to R and S is the set of all objects which can be possibly classified as X^c with respect to P and Q and it is denoted by $U_{R*S}(X)^c$. That is,

$U_{R*S}(X)^c= \bigcup_{x\in U}\{[x]: R(x)\cap X^c\neq\Phi$ or

$S(x)\cap X^c\neq\Phi$ }, where R[x] and S[x] denotes the equivalence class determined by x.

(iii) The multi star boundary region of X^c with respect to R and S is the set of all objects which can be classified neither as X^c nor as not X^c with respect to R and S and it is denoted by $B_{R*S}(X)^c$. That is, $B_{R*S}(X)^c = U_{R*S}(X)^c - L_{R*S}(X)^c$.

That is, $\tau_{R*S}(X)^c$ forms a topology on U called as the multi star granular nano topology on U with respect to X^c. We call $(U, \tau_{R*S}(X)^c)$ as the multi star granular nano topological space in terms of complement.

Example : 3.4 Let U={a, b, c, d, e} and U/R={{a},{b,c,d},{e}},U/S = {{a,b},{c, d, e}}} be two equivalence classes on U and X={a,b}$\subseteq$U, X^c={c,d,e}$\subseteq$U. Then $L_R(X)^c$ ={e},$U_R(X)^c$={b,c,d,e},$B_R(X)^c$={b,c,d} also we have $L_S(X)^c$={c,d,e}, $U_S(X)^c$={c,d,e}, $B_S(X)^c=\varnothing$. Therefore the nano topology $\tau_R(X)^c$={U,$\varnothing$,{e},{b,c, d,e},{b,c,d}} and $\tau_S(X)^c$={U,$\varnothing$,{c,d ,e}}. Now $L_{R*S}(X)^c$={e}, $U_{R*S}(X)^c$ ={b,c,d,e}, $B_{R*S}(X)^c$ = {b,c,d}. Hence the multi star granular nano topology $\tau_{R*S}(X)^c$= {U, $\varnothing$, {e}, {b,c,d,e}, {b,c,d}}.

Definition :3.5 Let U be a non empty finite set of objects called the universe and R, S be any two equivalence relations on U named as the indiscernibility relation.

Elements belonging to the same equivalence classes are said to be indiscernible with one another and $\tau_{R*S}(X-Y)$={U, $\varnothing$, $L_{R*S}(X-Y)$,$U_{R*S}(X-Y)$, $B_{R*S}(X-Y)$} where $L_{R*S}(X-Y)$, $U_{R*S}(X-Y)$, $B_{R*S}(X-Y)$ defined as follows. Let X-Y $\subseteq$U.

(i) The multi star lower approximation of X-Y with respect to R and S is the set of all objects which can be for certain classified as X-Y with respect to R and S and it is denoted by $L_{R*S}(X-Y)$. That is, $L_{R*S}(X-Y)$ $= \bigcup_{x\in U}\{[x] : R(x) \subseteq X-Y$ and $S(x) \subseteq X-Y\}$, where R[x] and S[x] denotes the equivalence class determined by x.

(ii) The multi star upper approximation of X-Y with respect to R and S is the set of all objects which can be possibly classified as X-Y with respect to R and S and it is denoted by $U_{R*S}(X-Y)$. That is

$U_{R*S}(X-Y) = \bigcup_{x\in U}\{[x] : R(x) \cap X-Y \neq \Phi$ or $S(x) \cap X-Y \neq \Phi\}$, where R[x] and S[x] denotes the equivalence class determined by x.

(iii) The multi star boundary region of X-Y with respect to R and S is the set of all objects which can be classified neither as X-Y nor as not X-Y with respect to P and Q and it is denoted by $B_{R*S}(X-Y)$. That is

$B_{R*S}(X-Y) = U_{R*S}(X-Y) - L_{R*S}(X-Y)$.

That is, $\tau_{R*S}(X-Y)$ forms a topology on U called as the multi star granular nano topology on U with respect to X^c. We call (U, $\tau_{R*S}(X-Y)$) as the multi star granular nano topological space in terms of Difference.

Example: 3.6 Let U={a, b, c, d} and U/R = {{a}, {b,c},{d}},U/S = {{a,b}, {c,d}}} be two equivalence relation on U and Let X={a,b,c}$\subseteq$U and Y={a}$\subseteq$U. Then $L_R(X-Y)$= {b,c}, $U_R(X-Y)$={b,c}, $B_R(X-Y)$

$=\varnothing$ also we have $L_S(X-Y)=\varnothing$, $U_S(X-Y)=\{a,b,c,d\}$, $B_S(X-Y)=\{a,b,c,d\}$.
Now $L_{R*S}(X-Y)=\varnothing$, $U_{R*S}(X-Y)=\{a,b, c,d\}$, $B_{R*S}(X-Y)=\{a,b,c,d\}$.

Hence the multi star granular nano topology $\tau_{R*S}(X-Y)= \{U, \varnothing,\{a,b,c,d\}\}$.

Proposition: 3.7 (De-Morgan's Law)

Let $(U, \tau_{R*S}(X\cup Y))$ be a multi star granular nano topological space with respect to X and Y. Let $X,Y\subseteq U$. Then
(a) $L_{R*S}(X\cup Y)^c= L_{R*S}(X)^c \cap L_{R*S}(Y)^c$.
(b) $U_{R*S}(X\cup Y)^c = U_{R*S}(X)^c \cap U_{R*S}(Y)^c$
(c) $L_{R*S}(X\cap Y)^c = L_{R*S}(X)^c \cup L_{R*S}(Y)^c$.
(d) $U_{R*S}(X\cap Y)^c = U_{R*S}(X)^c \cup U_{R*S}(Y)^c$.

Proposition: 3.8 If $(U, \tau_{R*S}(X\cup Y))$ is a multi star granular nano topological space with respect to X, Y are subsets of U, then
(i) $L_{R*S}(X\cap Y) =[L_R(X) \cap L_R(Y)] \cup [L_S(X)\cap L_S(Y)]$.
(ii) $U_{R*S}(X\cup Y) = [U_R(X) \cup U_R(Y)] \cap [U_S(X)\cup U_S(Y)]$.

Proof :
(i) For any $[x]\in L_{R*S}(X\cap Y)\Leftrightarrow R[x] \subseteq X\cap Y$ and $S[x] \subseteq X\cap Y \Leftrightarrow [x]\in L_R(X\cap Y)$ and $[x] \in L_S(X\cap Y) \Leftrightarrow [x] \in L_R(X\cap Y)\cup[x] \in L_S(X \cap Y) \Leftrightarrow [x] \in [L_R(X) \cap L_R(Y)]\cup [L_S(X)\cap L_S(Y)]$. Hence $L_{R*S}(X\cap Y)=[L_R(X)\cap L_R(Y)] \cup [L_S(X) \cap L_S(Y)]$.

(ii) For any $[x]\in U_{R*S}(X\cup Y) \Leftrightarrow R[x] \cap (X \cup Y)\neq\varnothing$ or $S[x]\cap(X\cup Y) \neq$ □$\Leftrightarrow [x]\in U_R(X \cup Y)$ or $[x]\in U_Q(X\cup Y)\Leftrightarrow[x] \in U_R(X\cup Y)\cap [x] \in U_S(X\cup Y) \Leftrightarrow [x]\in[U_R[X] \cup U_R(Y)]\cap [x] \in[U_S[X] \cup U_S[Y]$. Hence $U_{R*S}(X\cup Y) = [U_R(X) \cup U_R(Y)] \cap [U_S(X) \cup U_S(Y)]$.

Proposition : 3.8 Let U be universe and $X\subseteq U$. Let $\tau_{R*S}(X)^c$ be a multi star granular nano topology on U with respect to X^c. Then
(i) $L_{R*S}(X)^c = [U_{R*S}(X)]^c$
(ii) $U_{R*S}(X)^c = [L_{R*S}(X)]^c$.
(iii) $B_{R*S}(X)^c = B_{R*S}(X)$.

Proof:
(i) For any $[x] \in L_{R*S}(X)^c$ then $\$[x]\in L_{R*S}(X)^c$ which implies that $R[x] \subseteq X^c$ and $S[x] \subseteq X^c$. So that we have $R[x] \cap X^c \neq$ □ and $S[x]\cap X^c \neq \varnothing$. Therefore we have $[x] \in U_{R*S}(X^c)$. We can obtain that $[x]\in [U_{R*S}(X)]^c$. Hence $L_{R*S}(X)^c = [U_{R*S}(X)]^c$.
(ii) If $L_{R*S}(X)= [U_{R*S}(X^c)]^c$. So it can be obtain that $[L_{R*S}(X)]^c = U_{R*S}(X)^c$. Hence $U_{R*S}(X)^c =[L_{R*S}(X)]^c$.
(iii) From the definition 3.2 have boundary region of multi star granular nano topology and the duality of $L_{R*S}(X)$ and $U_{R*S}(X)$, we know that

$$B_{R*S}(X^c)=[U_{R*S}(X)]^c-[L_{R*S}(X)]^c.$$

Then$[L_{R*S}(X)]^c-[U_{R*S}(X)]^c$. So it can be obtain that $[U- L_{R*S}(X)]-[U-U_{R*S}(X)]$, which implies that $U_{R*S}(X)-L_{R*S}(X)$. Finally we get $B_{R*S}(X)$. Hence $B_{R*S}(X^c)=B_{R*S}(X)$.

Proposition:3.9 If $(U,\tau_{R*S}(X\cup Y))$ $(\tau_{R*S}(X\cap Y)$resp.,) is a multi star granular nano topological space with respect to X, Y are subsets of U, then
(i) $L_{R*S}(X\cup Y)^c= [L_{R*S}(X) \cup L_{R*S}(Y)]^c$
(ii) $L_{R*S}(X\cap Y)^c= [L_{R*S}(X) \cap L_{R*S}(Y)]^c$
(iii)$U_{R*S}(X \cup Y)^c=[U_{R*S} (X)\cup U_{R*S} (Y)]^c$

(iv)$U_{R*S}(X\ Y)^c= [U_{R*S}(X)\ U_{R*S}\ (Y)\ ]^c$.

Proof:

(i) For any $[x] \in L_{R*S}(X\ Y)^c$ also we have$[x] \in[L_{R*S}(X) \cup L_{R*S}(Y)]^c$ and $[x] \in [L_{R*S}(X)]^c \cap[L_{R*S}(Y)]^c$. Then by definition 3.2 we have $R[x] \subseteq (X\cap Y)^c$ or $S[x] \subseteq (X\cap Y)^c$ and also we have $R[x] \subseteq X^c$ and $R[x] \subseteq Y^c$ hold at the same time and $S[x]\ X^c$ and $S[x]\ Y^c$ hold at the same time. Therefore we have $R[x]\subseteq X^c$ and $S[x]\subseteq X^c$ also $R[x] \subseteq Y^c$ and $S[x] \subseteq Y^c$ hold at the same time, that is to say that $[x] \in L_{R*S}(X^c)$ and $[x] \in L_{R*S}(Y^c)$ which implies that $[x] \in L_{R*S}\ (X^c) \cup L_{R*S}(Y^c)$. Therefore by proposition 3.9 we have $[x]\in U_{R*S}(X^c)\cap U_{R*S}(Y^c)$. We can obtain that $[L_{R*S}(X)\cup L_{R*S}(Y)]^c=U_{R*S}(X^c) \cap U_{R*S}(Y^c)$. Hence $L_{R*S}(X\cup Y)^c = [L_{R*S}(X)\cup L_{R*S}(Y)]^c= U_{R*S}(X^c)\cap U_{R*S}(Y^c)$.

(ii) Similarly, the proof is proved by proof(i).

(iii) For any $[x] \in U_{R*S}(X\cup Y)^c$ we have $[x] \in [U_{R*S}(X) \cup U_{R*S}(Y)]^c$ and also $[x] \in [U_{R*S}(X)]^c \cap [U_{R*S}(Y)]^c$. Then by definition 3.2 we have $R[x] \cap X^c\neq\phi$ or $S[x] \cap X^c\neq\phi$ hold at the same time or $R[x] \cap Y^c\neq\phi$ or $S[x] \cap Y^c \neq\phi$ hold at the same time, which implies that $R[x]\cap (X\cup Y)^c\neq\phi$ or $S[x] \cap (X\cup Y)^c \neq\phi$. we can say that $[x] \in U_{R*S}(X\ Y)^c$, which implies that $[x] \in [U_{R*S}(X)]^c \cup [U_{R*S}(Y)]^c$. Therefore by proposition 3.9 we have $[x] \in L_{R*S}(X^c)] \cap [L_{R*S}(Y^c)]$. Here, we can obtain that $[U_{R*S}(X)\cup U_{R*S}(Y)]^c = L_{R*S}(X)^c\cap L_{R*S}(Y)^c$.

Hence $U_{R*S}(X\cup Y)^c= [U_{R*S}(X) \cup U_{R*S}(Y)]^c =L_{R*S}(X^c) \cap U_{R*S}(Y^c)$.

(iv) Similarly, the proof is proved by proof (iii).

Proposition:3.10 Let $(U,\tau_{R*S}(X\cup Y))$ be a multi star granular nano topological space with respect to X and Y. Let $X,Y \subseteq U$. Then

(i) $L_{R*S}(X\cup Y)^c=U_{R*S}(X^c) \cap U_{R*S}(Y^c)$.

(ii)$L_{R*S}(X\cap Y)^c=U_{R*S}(X^c) \cup U_{R*S}(Y^c)$.

(iii)$U_{R*S}(X\cup Y)^c =L_{R*S}(X^c) \cap L_{R*S}(Y^c)$.

(iv)$U_{R*S}(X\cap Y)^c= L_{R*S}(X^c) \cup L_{R*S}(Y^c)$.

IV. *Properties of Approximations In MSGNT and MGNT :*

In this section, we introduce the approximation concept represented by subsets of the universe. Comparison between the multi star granular nano topology (MSGNT) and multi granular nano topology (MGNT) its based on intersection of any two equivalence class defined on U.

Definition : 4.1 Let U be the universe and U/R, U/S be two equivalence classes defined on U. Then intersection of two equivalence classes is defined on $U/R\cap S$ as $U/R\cap S=\{R\cap S:R\in U/R,S\in U/S$ and $R\cap S \neq\phi$.

Definition :4.2 Let U be the universe of objects and R,S be any two equivalence classes on U and $\tau_{R\cap S}(X \cup Y) =\{U,\phi, L_{R\cap S}(X\cup Y),U_{R\cap S}(X\cup Y), B_{R\cap S}(X\ Y)\}$where $L_{R\cap S}(X\cup Y),U_{R\cap S}(X\cup Y),B_{R\cap S}(X\cup Y)$ defined as follows.

Let $X\ Y\subseteq U$.

(i) The lower approximation of $X\cup Y$ with respect to $R\cap S$ is denoted by $L_{R\cap S}(X\cup Y)$. That is $L_{R\cap S}(X\cup Y)= \bigcup_{x\in U}\{[x]\in U/R\cap S:[x]\subseteq X\cup Y\}$.

(ii) The upper approximation of $X \cup Y$ with respect to $R \cap S$ is denoted by $U_{R \cap S}(X \cup Y)$.

That is $U_{R \cap S}(X \cup Y) = \bigcup_{x \in U} \{[x] \in U/R \cap S : [x] \cap X \cup Y \neq \phi\}$.

(iii) The boundary region of $X \cup Y$ with respect to $R \cap S$ is denoted by $B_{R \cap S}(X \cup Y)$. That is $B_{R \cap S}(X \cup Y) = U_{R \cap S}(X \cup Y) - L_{R \cap S}(X \cup Y)$.

That is $\tau_{R \cap S}(X \cup Y)$ forms a topology on U called as the multi granular nano topology in terms of intersection on U with respect to $X \cup Y$.

Example 4.3: Let U = {a, b, c, d, e} with U/R ={{a,b}, {c,d},{e}} and U/S ={{a,e},{b,c,d}} be any two equivalence relations defined on U. Let X = {a,d},Y ={b,c} and $X \cup Y$= {a,b,c,d}}. Then $U/R \cap S$ = {{a}, {b}, {c,d},{e}}, $L_{R \cap S}(X \cup Y)$ ={a,b, c,d},$U_{R \cap S}(X \cup Y)$={a,b,c,d},$B_{R \cap S}(X Y) = \emptyset$. Then $\tau_{R \cap S}(X \cup Y) = \{U, \emptyset, \{a,b,c,d\}\}$.

Proposition 4.4: Let (U, $\tau_{R*S}(X \cup Y)$) be a multi star granular nano topological space with respect to X and Y. Let $X, Y \subseteq U$. Then

(i) $L_{R \cap S}(X \cup Y) \subseteq X \cup Y$

(ii) $U_{R \cap S}(X \cup Y) \subseteq X \cup Y$

(iii) $L_{R \cap S}(X \cup Y)^c = [U_{R \cap S}(X \cup Y)]^c$

where $(X \cup Y)^c$ is the complement of $(X \cup Y)$.

(iv)$U_{R \cap S}(X \cup Y)^c = [L_{R \cap S}(X \cup Y)]^c$

where $(X \cup Y)^c$ is the complement of $(X \cup Y)$.

(v)$L_{R \cap S}(\emptyset) = \emptyset$ and $U_{R \cap S}(\emptyset) = \emptyset$.

(vi)]$L_{R \cap S}(U) = U$ and $U_{R \cap S}(U) = U$.

(vii)$L_{R \cap S}(X \cup Y) = L_{S \cap R}(X \cup Y)$ and $U_{R \cap S}(X \cup Y) = U_{S \cap R}(X \cup Y)$.

Proposition 4.4: Let (U,R) be an approximation space and $U/R \cap S$ be equivalence relations defined on U. Let $X, Y \subseteq U$. Then

(i) $L_{R \cap S}(L_{R \cap S}(X \cup Y)) = L_{R \cap S}(X \cup Y)$

(ii) $U_{R \cap S}(U_{R \cap S}(X \cup Y)) = U_{R \cap S}(X \cup Y)$.

(iii) $L_{R \cap S}(X \cup Y) \supseteq L_{R \cap S}(X) \cup L_{R \cap S}(Y)$.

(iv) $U_{R \cap S}(X \cup Y) \supseteq U_{R \cap S}(X) \cap U_{R \cap S}(Y)$.

Proposition 4.5: Let $(U, \tau_{R*S}(X \cup Y))$ $(\tau_{R \cap S}\}(X \cup Y)$ resp.) be a multi star granular nano topological space with respect to X and Y. Let $X, Y \subseteq U$. Then

(i)$L_{R*S}(X \cup Y) \subseteq L_{R \cap S}(X \cup Y)$

(ii)$U_{R*S}(X \cup Y) \subseteq U_{R \cap S}(X \cup Y)$

Proof:

(i) For any $[x] \in L_{R*S}(X \cup Y)$ from definition it follows that $[x] \in R[x]$ and $[x] \in S[x]$. Hence $[x] \in R[x] \cap S[x]$. But $R[x] \cap S[x] \in U/R \cap S$ for any $x \in U$, $L_{R \cap S}(X \cup Y) = \bigcup_{x \in U} \{[x] \in U/R \cap S : [x] \subseteq X \cup Y\}$ from the definition, therefore $[x] \in L_{R \cap S}(X \cup Y)$. Therefore $L_{R*S}(X \cup Y) \subseteq L_{R \cap S}(X \cup Y)$.

(ii) Applying proof(i) we have that $U_{R \cap S}(X \cup Y) = [L_{R \cap S}(X \cup Y)^c]^c \subseteq [L_{R*S}(X \cup Y)^c]^c = U_{R \cap S}(X \cup Y)$. Hence $U_{R*S}(X \cup Y) \subseteq U_{R \cap S}(X \cup Y)$.

V. *Real life Application :*

In this part, we concern the Multi star granular nano topological spaces to discover the information system (U,A), where U is the universe and A is a finite set of attributes divided into conditional attribute, decision attribute in MSGNT and then investigate the nano accuracy measure, nano degree of dependence by capture the "summary of reviewers reports for eight papers" details in a decision table.

Definition 5.1: Let (U,AT $\cup$ D,V) be an complete information system, where U is a non empty finite set of objects, A is a finite set of attributes and A is divided into a set C of conditional attributes and a set D of decision attributes where AT D$\neq\emptyset$ and V is regarded as the domain of all attributes[12].

Definition 5.2: Let (U,$\tau_{R\cap S}$(X$\cup$Y)) be a intersecti -on of multi granular nano topological space and X,YU, then the nano accuracy measure of XY in terms of union is defined as

$$\delta_{R\cap S}(X\cup Y)=\frac{B_{R\cap S}(X\cup Y)}{U_{R\cap S}(X\cup Y)}.$$

Definition 5.3: Let (U, τ_{R*S}(X$\cup$Y)) be an multi star granular nano topological space. Let X,Y$\subseteq$U and all decision classes U/D ={D_1, D_2, D_3,.....,D_k} induced by decision attribute D and A is divided into a two set C of condition attribute, decision attributes D, then the approximation of A is said to be nano degree of dependence based on upper approximation and is definedas

$$\gamma^{'}[R*S,D]=\frac{1}{|U|}[|U_{R*S}(D_1)|+$$

$$|U_{R*S}(D_2)|+........+|U_{R*S}(D_k)|$$

Example 5.4: In this section, we have calculated the nano accuracy measure and nano degree of dependency based on upper approximation in multi star granular nano topology and also essential operations of union and intersection in MSGNT by taking a summary of reviewer's reports for eight papers submitted to a journal data. consider the following table giving information about the every row consist of a "summary of reviewer's reports for 8 papers" evaluated by means of condition attributes is an "Originality", "Presentation", "Technical soundness" and the decision attributes is an overall evaluation.

In the sequel, O, P, T and will stand for Originality, Presentation, Technical Soundness and Decision respectively.

The domains are as follows: V_O = {Excellent, Good, Fair};V_P={Excellent, Fair},V_T = {Good, Fair}. The decision part which has only three possible values are accept, revision and reject.

The columns of the table represent the essential factors for initializing the summary of reviewer's, and the rows represent the individual project. The entries in the table are the attribute values.

Project	O	P	T	Decision
x_1	excellent	excellent	good	Accept
x_2	good	fair	good	revision
x_3	excellent	fair	fair	revision
x_4	fair	excellent	fair	Reject
x_5	fair	fair	fair	Reject
x_6	good	fair	fair	Reject
x_7	excellent	excellent	good	revision
x_8	excellent	fair	good	Accept

Here U= {x_1, x_2, x_3,x_4, x_5, x_6, x_7, x_8}, the set of eight projects and A = {Originality, Presentation, Technical Soundness, Decision part} be the set of attribute. From the table, we can find that

$U/O=\{\{x_1,x_3,x_7,x_8\},\{x_2,x_6\},\{x_4\},\{x_5\}\}$.
$U/P=\{\{x_1,x_4,x_7\},\{x_3,x_6,x_8\},\{x_2,x_5\}\}$
$U/O\cap P=\{\{x_1,x_7\}, \{x_3,x_8\}, \{x_2\}, \{x_4\}, \{x_5\}, \{x_6\}\}$. Let us take $X=\{x_2, x_6,x_5\}\subseteq U$ and $Y=\{x_5\}\subseteq U$ and $X\cup Y=\{x_2,x_5, x_6\}$. Then $L_{O*P}(X\cup Y)=\{x_2\}$, $U_{O*P}(X\cup Y)=\{x_2,x_3,x_4,x_5, x_6,x_8\}$, $B_{O*P}(\cup Y)=\{x_3,x_4, x_5,x_6,x_8\}$, $L_O(X\cup Y)=\{x_2,x_6\}$, $U_O(X\cup Y)=\{x_2, x_4,x_5, x_6\}$, $B_O(X\cup Y)=\{x_4, x_5,x_6\}$, $L_P(X\cup Y)=\{x_2,x_5\}$, $U_P(X\cup Y)=\{x_2,x_3, x_5,x_6, x_8\}$, $B_{O*P}(X\cup Y)=\{x_3,x_6,x_8\}$, $L_{O\cap P}(X\cup Y)=\{x_2,x_5,x_6\}$, $U_{O\cap P}(X\cap Y)=\{x_2, x_5,x_6\}$, $B_{O\cap P}(X\cap Y)=$□. Hence we can say that
$L_{O*P}(X\cup Y)\subseteq L_{O\cap P}(X\cup Y)\subseteq X\cup Y\subseteq U_{O\cap P}(X\cup Y)\subseteq U_{O*P}(X\cup Y)$.

To find the nano accuracy measure:

Now we find the nano accuracy measures of $X\cup Y=\{x_2,x_5,x_6\}$ in both single granulation and multi granulation as follows:

$$\delta_O(X\cup Y)=\frac{B_O(X\cup Y)}{U_O(X\cup Y)}=\frac{2}{4}.$$

$$\delta_P(X\cup Y)=\frac{B_P(X\cup Y)}{U_P(X\cup Y)}=\frac{3}{5}.$$

$$\delta_{O\cap P}(X\cup Y)=\frac{B_{O\cap P}(X\cup Y)}{U_{O\cap P}(X\cup Y)}=0.$$

$$\delta_{O*P}(X\cup Y)=\frac{B_{O*P}(X\cup Y)}{U_{O*P}(X\cup Y)}=\frac{5}{6}.$$

Hence by comparing the results we can conclude that

$$\delta_{O*P}(X\cup Y)\geq\delta_O(X\cup Y)\geq\delta_{O\cap P}(X\cup Y)$$

$$\delta_{O*P}(X\cup Y)\geq\delta_P(X\cup Y)\geq\delta_{O\cap P}(X\cup Y)$$

To compute the degree of dependence based on upper approximation :

Now, we compute the degree of dependence based on upper approximation of $X\cap Y=\{x_2,x_5,x_6\}$. From the table, we have $U/D=\{D_A,D_R,D_r\}$, where $D_A=\{x_1,x_8\}$ and $D_R=\{x_4,x_5,x_6\}$, $D_r=\{x_2, x_3,x_7\}$ and $U/D=\{\{x_1,x_8\},\{x_4,x_5,x_6\},\{x_2,x_3,x_7\}\}$.

From the table

$U_O(D_A)=\{x_1,x_3,x_7,x_8\}$
$U_P(D_A)=\{x_1,x_3,x_4,x_6,x_7,x_8\}$
$U_{O*P}(D_A)=\{x_1,x_3,x_4,x_6,x_7,x_8\}$
$U_O(D_R)=\{x_2,x_4,x_5,x_6\}$
$U_P(D_R)=U$
$U_{O*P}(D_R)=U$
$U_O(D_r)=\{x_1,x_2,x_3,x_6,x_7,x_8\}$

$U_P(D_r)=U$
$U_{O*P}(D_r)=U$
So, we have

$$\gamma'[O,D]=\frac{1}{|U|}[|U_O(D_A)|+|U_A(D_R)|+|U_A(D_r)|=\frac{7}{4}$$

$$\gamma'[P,D]=\frac{1}{|U|}[|U_P(D_A)|+|U_P(D_R)|+|U_P(D_r)|=\frac{22}{8}$$

$$\gamma'[O*P,D]=\frac{1}{|U|}[|U_{O*P}(D_A)|+|U_{O*P}(D_R)|+|U_{O*P}(D_r)|=\frac{22}{8}$$

$$\gamma'[P*O,D]=\frac{1}{|U|}[|U_{P*O}(D_A)|+|U_{P*O}(D_R)|+|U_{P*O}(D_r)|=\frac{22}{8}$$

Hence, from the above it can be found that,

Observation

A comparative analysis of Nano topology and MSGNT is given based on nano accuracy measure and nano degree of dependency based on upper approximation and we consider the similar data set and analyse that the claim Muti star granular is always much better than that defined by using a single granulation[1-12].

Conclusion

The algebraic set theoretical operations approach to nano topological space is appropriate for studying the MSGNT.

The set approximations are defined by using multi equivalence on the universe and the single granulation it has been extended to be multi granulation[9-10].

A comparative analysis of "Nano topology" and "MSGNT" is given based on nano accuracy measure and nano degree of dependency based on upper approximation. Therefore, multiple granulation is always much better that defined by using a single granulation which is suitable for more precisely characterizing a target concept and problem solving according to user's requirements. These results can be used for further studies in approximation of classification and rule generation.

References

1. E.A. Abo-Tabl, "A Comparision of two kinds of definitions of rough approximation based on similarity relation", Information Sciences, *181*, 2587-2596 (2011).
2. A. Bargiela and W. Pedrycz, "Granular Computing An Introduction", Kluwer Academic Publishers, Boston, 73-89 (2002).
3. M.I. Ali, M. Shabir and M. Naz, "Algebraic Structures of Soft sets associated with new operations", Computers and mathematics with application, *61*, 2647-2654 (2011).
4. M. Lellis Thivagar and Carmel Richard, "On Nano Forms of Weakly Open sets", Internat. J. Math. and stat. Inv. Vol. *1*, No. 1, 31-37 (2013).
5. M. Lellis Thivagar and Carmel Richard, "On Nano Continuity in a Strong Form", International Journal of Pure and Applied Mathematics, Vol.*101*, No.5, 893-904 (2015).
6. J.Y. Liang and Z.Z. Shi, "The Information entropy, rough entropy and knowledge granulation in rough set theory", International Journal of Uncertainty, Fuzziness and Knowledge-Based systems, Vol{12}(1) 37-46 (2001).
7. D. Pei, "On definable concepts of rough set models", Informations Sciences, *177*, 4230-4239 (2007).
8. Z. Pawlak, "Rough set theory and Its applications", Journal of Telecommunication Information Technology, Vol *3*, 7-10 (2002).
9. Z. Pawlak, Rough Sets, "International journal of computer and Information Sciences", *11*, 341-356 (1982).
10. Y.H. Qian, J.Y. Liang, Y.Y. Yao and C.Y Dang, "A Multi-Granulation Rough set", Information Sciences, *180*, 949-970 (2010).
11. S. Salama, "Some topological properties of rough sets with tools for data mining", International Journal of Computer Science Issues, Vol. *8*, Issue 3, No.2, 588-595 (2011).
12. Y.Y. Yao, "A Comparative Study of formal concept analysis and rough set theory in data analysis", Rough sets and Current Trends in Computing, Vol. 3066 59-68 (2004).