ISSN 2231-346X (Print)
JUSPS-A Vol. 29(9), 388-400 (2017). Periodicity-Monthly
Section A
(Online) ISSN 2319-8044

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Mathematics
website:- www.ultrascientist.org
Estd. 1989

A Production fuzzy Inventory model for deteriorating items Defuzzified by Graded Mean Integration Representation Method

S. K. INDRAJITSINGHA[1], P.N. SAMANTA[2] and U.K. MISRA[3]

[1]DST INSPIRE fellow, P.G. Department of Mathematics, Berhampur University, Berhampur, Bhanja Bihar-760 007, Odisha (India)
[2]P.G. Department of Mathematics, Berhampur University, Berhampur, Bhanja Bihar-760 007, Odisha (India)
[3]Department of Mathematics, NIST, Golanthara, PalurHill, Pin-761 008, Berhampur, Odisha, (India)
Email Address of Corresponding Author: ski.rs.math@buodisha.edu.in
http://dx.doi.org/10.22147/jusps-A/290905

Acceptance Date 13th August, 2017, Online Publication Date 2nd September, 2017

Abstract

This paper develops a production inventory model for deteriorating items under fuzzy environment. Since holding cost, set-up cost, deterioration cost, deterioration rate, production rate, demand etc. are uncertain in nature; these are carried by pentagonal fuzzy numbers. Graded Mean Integration Representation (GMIR) method is used to defuzzify the total cost function. Numerical example is given to explore the theoretical results and made the comprehensive. Sensitivity analysis with different parameters on the optimal solution is carried to illustrate the effectiveness and behavior of the model.

Key words: Inventory Model, Pentagonal Fuzzy Number, Graded Mean Integration Representation Method, Defuzzification

AMS Classification No: 03E72

1. Introduction

Inventory means physical stock of goods. An inventory fulfills many important functions within the organization. In manufacturing and business operations, the inventory problems are common factors. Harris[8], developed the first inventory model in 1915 taking very few parameters. But, in real life inventory, there are many such parameters like demand, deterioration rate, holding cost, production rate, set-up cost etc. which are uncertain. For solving this type of randomness, researchers traditionally applied probability theory (Covert and

Philip[2], Ghare and Schrader[7]).

However, in certain situations, uncertainties are due to fuzziness, and such cases are dilated in the fuzzy set theory which was developed by Zadeh[30] in 1965. After the invention of fuzzy set theory, again in 1970, Zadeh and Bellman[31] developed a mathematical model on decision making in fuzzy environment. Hence most of researchers attracted towards the theory of fuzzy sets. Dubois and Prade[4] define some operations on fuzzy numbers. In 1991, Kaufmann and Gupta[14] provided an introduction to fuzzy arithmetic operations and Zimmerman[32] discussed the concept of the fuzzy set theory and its applications.

Some researchers started to apply fuzzy set theory in inventory management problems. Kacpryzk and Staniewski[13] proposed a model on long-term inventory policy-making through fuzzy-decision making model. In 1987, K.S Park[19] developed an economic order quantity model using fuzzy set theory. Yao and Lee[29] proposed a fuzzy inventory model with backorder for fuzzy order quantity. In 1996, Vujosevic *et. al*.[28] developed an EOQ formula when inventory cost is fuzzy. In 1997, Gen *et. al.*[6] introduce a fuzzy inventory control model. Chang[1] applied fuzzy triangular number in production inventory model. Syed and Aziz[27] in 2007 applied signed distance method to defuzzify the inventory model without shortage. In 2011, De and Rawat[3] developed a fuzzy inventory model without shortages using triangular fuzzy number. In 2012, Jaggi *et. al.*[12] explored a fuzzy inventory model for deteriorating items with time-varying demand and shortages in which parameters are treated as triangular fuzzy number. They defuzzified this model by different defuzzification methods. In 2013, Dutta and Kumar[5] developed a fuzzy inventory model for deteriorating items with shortages under fully backlogged condition and defuzzified by this method by different defuzzification methods. Singh and Singh[26] proposed an integrated inventory model from the perspective of a single vendor and multi-buyers for deteriorating items under fuzzy environment and inflation. In that development they consider all costs and inflation as trapezoidal fuzzy number and defuzzified by graded mean integration representation method.

A fuzzy inventory model for deteriorating items with price dependent demand rate was developed by Maragatham and Lakshmidevi[16] in 2014. In which they consider demand as trapezoidal fuzzy number and defuzzified by applying signed distance method. In 2014, Ranganathan and Thirunavukarasu[20] discussed an inventory control model for constant deterioration and logarithmic demand rate under fuzzy environment. Where they defuzzified the total cost function by graded mean integration representation method. Nagar and Surana[18] developed an inventory model for deteriorating items with fluctuating demand using inventory parameters as pentagonal fuzzy numbers. They defuzzified that the model by graded mean integration representation method. In 2015, Kumar and Rajput[15] proposed a fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging in which the demand rate , deterioration rate , backlogging rate are assumed as a triangular fuzzy numbers. They defuzzified the total cost function of that model by signed distance method and centroid method. Ranganathan and Thirunavukarasu[21] formulated a fuzzy inventory model under immediate return for deficient items in which they used triangular fuzzy number. Mishra *et. al*.[17] proposed an inventory control model of deteriorating items where the deteriorating rate, deteriorating cost, carrying cost and shortages are taken as trapezoidal fuzzy numbers. They defuzzified the model by using graded mean integration representation method. Indrajitsingha *et. al*.[10] studied a fuzzy economic production quantity model with time dependent demand rate in which demand cost and holding cost are taken as triangular fuzzy numbers and defuzzified the total cost function by using different defuzzification methods.

In 2016, Indrajitsingha *et. al*.[11] extended a fuzzy inventory model with shortages under fully backlogged in where they defuzzified the total cost function by using signed distance method. Raula *et. al*.[22] proposed a fuzzy inventory model for constant deteriorating items by using graded mean integration representation method

in which inventory parameters treated as hexagonal fuzzy numbers. Indrajitsingha *et. al.*[9] developed a fuzzy economic production quantity model with time dependent demand rate in which the parameters treated as pentagonal fuzzy numbers. They defuzzified the total cost functions by using signed distance method. In 2016, Sahoo *et. al.*[24] introduced an inventory model with exponential demand and time-varying deterioration in fuzzy approach. Where they used trapezoidal fuzzy numbers as holding cost, deterioration and purchase cost and total cost function was defuzzified by using graded mean integration representation method. Sen *et. al.*[25] considered a fuzzy inventory model for deteriorating items based on different defuzzification techniques in which parameters treated as triangular fuzzy numbers. Recently, Sahoo *et. al.*[23] developed a fuzzy inventory model with time dependent demand rate without shortages using pentagonal fuzzy number.

2. Objective and Organizations:

In the present model the cycle length T is divided in to two parts. In the period $(0,T_1)$, the roduction item going on with a constant rate and due to the demand the inventory decreases. In the period (T_1, T), the production is stopped with the same demand rate and the demand decreases to zero. We assumed that the demand rate is constant in both periods. In this paper we developed a fuzzy inventory model for deteriorating items in which all parameters are treated as pentagonal fuzzy number. Single inventory is used. The total cost function is defuzzified by using the graded mean integration representation (GMIR) method.

This paper is organized as follows: In sect. 3, definitions and preliminaries are given. In sect. 4, assumptions and notations of the proposed model are given. Mathematical model in Crisp and Fuzzy sense is formulated in sect. 5. Numerical example is illustrated in sect. 6 to support the proposed model. Sensitivity analysis is carried out by using Matlab R2011b software in sect. 7 followed by conclusion and future of scope.

3. Definitions and Preliminaries:

In order to establish the model we require the following definitions:

Definition 3.1 **(Fuzzy Set)** Let X be a space of points with a generic element x of X. Let $\mu: X \longrightarrow [0,1]$ be such that for every $x \in X$, $\mu(x)$ is a real number in the interval [0,1], usually called 'grade of membership'. We define a fuzzy set $\tilde{A}$ in X as the set of points $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$.

Definition 3.2 A fuzzy number $\tilde{A} = (a, b, c, d, e)$ where $a < b < c < d < e$ and defined on *R,* is called pentagonal fuzzy number if its membership function is

$$\mu_{\tilde{A}} = \begin{cases} L_1(x) = \dfrac{x-a}{b-a}, a \le x \le b \\ L_2(x) = \dfrac{x-b}{c-b}, b \le x \le c \\ 1, \qquad\qquad x = c \\ R_1(x) = \dfrac{d-x}{d-c}, c \le x \le d \\ R_2(x) = \dfrac{e-x}{e-d}, d \le x \le e \\ 0, \qquad Otherwise \end{cases}$$

α-cut of Pentagonal Fuzzy Number:

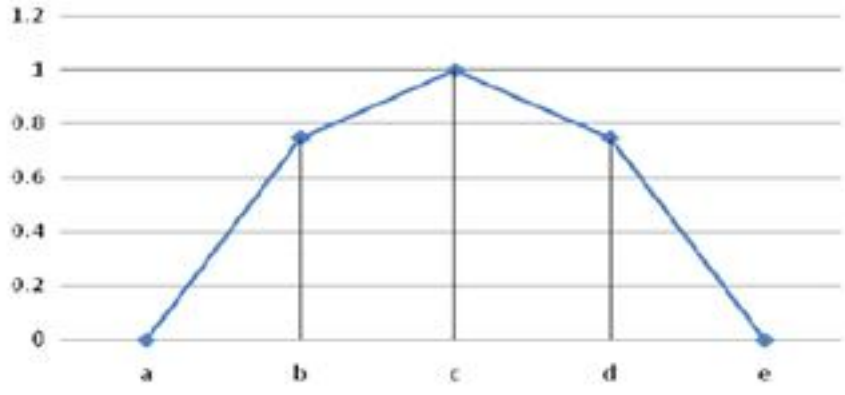


Fig.1 Pentagonal Fuzzy Number

Definition 3.3: If $\tilde{A} = (a, b, c, d, e)$ is a pentagonal fuzzy number then the graded mean representation method of $\tilde{A}$ is defined as

$$P(\tilde{A}) = \frac{\int_0^{w_A} \alpha \left(\frac{L^{-1}(\alpha) + R^{-1}(\alpha)}{2} \right) d\alpha}{\int_0^{w_A} \alpha d\alpha}$$

with $0 < \alpha \leq w_A$ and $0 < w_A \leq 1$

$$P(\tilde{A}) = \frac{1}{12}[a + 3b + 4c + 3d + e]$$

4. Assumptions and Notations:

Throughout the manuscripts, we make use of the following assumptions:

i. Single inventory will be used.
ii. Items are produced and added to the inventory.
iii. The lead time is zero.
iv. No shortages are allowed.
v. Replenishment is instantaneous.
vi. Time horizon is finite.
vii. The production rate is proportional to demand rate.
viii. The production rate is always greater than demand rate.
ix. There is no repair of deteriorated items occurring during the cycle.
x. Neglecting the higher power of θ.

Notations:

S_c = Set-up cost
θ = Deterioration rate independent of time, $0 < \theta \leq 1$
T = Cycle length

P_r = Production rate

h_c = Holding cost per unit per unit item

d_c = Deterioration cost per unit per unit time

D = Demand rate is constant

T_1 = Duration of production

$I_1(t)$ = Inventory level at time t, $0 \le t \le T_1$

$I_1(t)$ = Inventory level at time t, $T_1 \le t \le T$

T_c = Total cost per unit time

$\tilde{S}_c$ = Fuzzy set-up cost

$\tilde{\theta}$ = Fuzzy deterioration

$\tilde{P}_r$ = Fuzzy production arte

$\tilde{h}_c$ = Fuzzy holding cost per unit per unit time

$\tilde{d}_c$ = Fuzzy deterioration cost per unit per unit time

$\widetilde{D}$ = Fuzzy demand rate

$\tilde{T}_c$ = Total fuzzy inventory cost of

$\widetilde{TC}_{cg}$ = Defuzzified value of $\widetilde{TC}$ by applying GMIR method

5. *Mathematical Formulation :*

The status of inventory is shown in Fig. 2 as follows

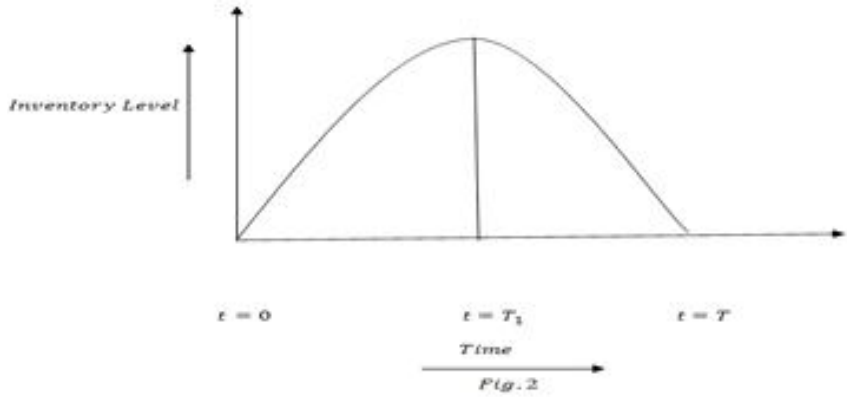


Fig. 2

A machine starts with a constant rate of production and stopped at time T_1, and initially, inventory level increases up to time T_1 with a constant rate P_r. When the production stopped; the inventory level decreases and reduces to zero due to the combined effect of demand and the deterioration. Hence the rate of change of inventory is governed by the differential equations represented as following:

(5.1) $$\frac{dI_1(t)}{dt} = P_r - \{D + \theta I_1(t)\}, \quad 0 \le t \le T_1$$

and

(5.2) $$\frac{dI_2(t)}{dt} = -\{D + \theta I_2(t)\}, \quad T_1 \le t \le T$$

Solution of (5.1) and (5.2) with the condition is

(5.3) $$I_1(t) = \frac{1}{\theta}(P_r - D)(1 - e^{-\theta t})$$

and

(5.4) $$I_2(t) = \frac{D}{\theta}(e^{\theta(T-t)} - 1)$$

Now, we find T_1 by using $I_1(T_1) = I_2(T_1)$

$$\frac{1}{\theta}(P_r - D)(1 - e^{-\theta t}) = \frac{D}{\theta}(e^{\theta(T-t)} - 1)$$

(5.5) $$\Rightarrow T_1 = \frac{1}{\theta}\log\left\{1 + \frac{D}{P_r}(e^{\theta T} - 1)\right\}$$

The total cost is calculated by considering set-up cost, holding cost and deterioration cost.

1. Set-up cost = S_c
2. Holding cost per cycle = $h_c\left[\int_o^{T_1} I_1(t)dt + \int_{T_1}^{T} I_2(t)dt\right]$

$$= \frac{h_c}{\theta}[P_r T_1 - DT]$$

3. Deterioration cost per cycle = $d_c\left[\int_o^{T_1} \theta I_1(t)dt + \int_{T_1}^{T} \theta I_2(t)dt\right]$

$$= d_c(P_r T_1 - DT)$$

Total cost of the system per unit time is given by

(5.6) $$T_c = \frac{1}{T}[Setup\ cost + Holding\ cost + Deterioration\ cost]$$

$$= \frac{S_c}{T} + \frac{(P_r T_1 - DT)}{\theta T}(h_c + d_c\theta)$$

Using (5.5) and the assumption (x) we get

$$= \frac{1}{T}\left[S_c + \frac{1}{2}(h_c + d_c\theta)DT^2 - \frac{1}{2}(h_c + d_c\theta)\frac{D^2T^2}{2}\right]$$

Fuzzy Model:

We consider the model in fuzzy approach. Due to vagueness, it is not easy to define all the parameters precisely. Accordingly we assume the parameters S_c, h_c, d_c, θ, P_r and D in fuzzy environment.

Suppose $\tilde{S}_c = (S_1, S_2, S_3, S_4, S_5,)$, $\tilde{h}_c = (h_1, h_2, h_3, h_4, h_5)$, $\tilde{d}_c = (d_1, d_2, d_3, d_4, d_5)$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$, $\tilde{P}_r = (P_1, P_2, P_3, P_4, P_5)$ and $\tilde{D} = (D_1, D_2, D_3, D_4, D_5)$ are as pentagonal fuzzy number.

Total cost of the system per unit time in fuzzy environment is given by

(5.7) $\tilde{T}_c = \frac{1}{T}\left[\tilde{S}_c + \frac{1}{2}(\tilde{h}_c + \tilde{d}_c\tilde{\theta})\tilde{D}T^2 - \frac{1}{2}(\tilde{h}_c + \tilde{d}_c\tilde{\theta})\frac{\tilde{D}^2T^2}{\tilde{P}_r}\right]$

We defuzzify the fuzzy total cost $\tilde{T}_c$ by Graded Mean Integration Representation Method. Thus the defuzzified value of $\tilde{T}_c$ is

$\tilde{T}_{cg} = \frac{1}{12}[\tilde{T}_{cg1} + 3\tilde{T}_{cg2} + 4\tilde{T}_{cg3} + 3\tilde{T}_{cg4} + \tilde{T}_{cg5}]$

Where

$\tilde{T}_{cgi} = \frac{1}{T}\left[S_i + \frac{1}{2}(h_i + d_i\theta_i)D_iT^2 - \frac{1}{2}(h_i + d_i\theta_i)\frac{{D_i}^2T^2}{2}\right], for\ i = 1,2,3,4,5$

To minimize the total cost function per unit time $\tilde{T}_{cg}$, the optimum value of T can be obtained by solving the differential equation

$\frac{d\tilde{T}_{cg}}{dT} = 0, \frac{d^2\tilde{T}_{cg}}{dT^2} > 0$

i.e. $\frac{d\tilde{T}_{cg}}{dT} = \frac{1}{12}\left[\frac{d}{dT}\tilde{T}_{cg1} + 3\frac{d}{dT}\tilde{T}_{cg2} + 4\frac{d}{dT}\tilde{T}_{cg3} + 3\frac{d}{dT}\tilde{T}_{cg4} + \frac{d}{dT}\tilde{T}_{cg5}\right] = 0$

Now

$\frac{d\tilde{T}_{cgi}}{dT} = -\frac{S_i}{T^2} + \left[\frac{1}{2}(h_i + d_i\theta_i)D_i - \frac{1}{2}(h_i + d_i\theta_i)\frac{{D_i}^2}{P_i}\right] for\ i = 1,2,3,4,5$

Thus

$$\frac{d\tilde{T}_{cg}}{dT} = \frac{1}{12}\left[-\frac{S_1 + 3S_2 + 4S_3 + 3S_4 + S_5}{T^2} + \left\{\begin{matrix}\frac{1}{2}(h_1 + d_1\theta_1)D_1 + \frac{3}{2}(h_2 + d_2\theta_2)D_2 \\ +2(h_3 + d_3\theta_3)D_3 + \frac{3}{2}(h_4 + d_4\theta_4)D_4 \\ +\frac{1}{2}(h_5 + d_5\theta_5)D_5\end{matrix}\right\} - \left\{\begin{matrix}\frac{1}{2}(h_1 + d_1\theta_1)\frac{{D_1}^2}{P_1} + \frac{3}{2}(h_2 + d_2\theta_2)\frac{{D_2}^2}{P_2} + 2(h_3 + d_3\theta_3)\frac{{D_3}^2}{P_3} \\ +\frac{3}{2}(h_4 + d_4\theta_4)\frac{{D_4}^2}{P_4} + \frac{1}{2}(h_5 + d_5\theta_5)\frac{{D_5}^2}{P_5}\end{matrix}\right\}\right]$$

Since $\frac{d\tilde{T}_{cg}}{dT} = 0$, we get

$$T^2 = \frac{2(S_1 + 3S_2 + 4S_3 + 3S_4 + S_5)}{\left\{\begin{matrix}(h_1 + d_1\theta_1)D_1\left(1 - \frac{D_1}{P_1}\right) + 3(h_2 + d_2\theta_2)D_2\left(1 - \frac{D_2}{P_2}\right) + 4(h_3 + d_3\theta_3)D_3\left(1 - \frac{D_3}{P_3}\right) \\ +3(h_4 + d_4\theta_4)D_4\left(1 - \frac{D_4}{P_4}\right) + (h_5 + d_5\theta_5)D_5\left(1 - \frac{D_5}{P_5}\right)\end{matrix}\right\}}$$

$$T = \sqrt{\frac{2(S_1 + 3S_2 + 4S_3 + 3S_4 + S_5)}{\left\{\begin{array}{c}(h_1 + d_1\theta_1)D_1\left(1 - \frac{D_1}{P_1}\right) + 3(h_2 + d_2\theta_2)D_2\left(1 - \frac{D_2}{P_2}\right) + 4(h_3 + d_3\theta_3)D_3\left(1 - \frac{D_3}{P_3}\right) \\ +3(h_4 + d_4\theta_4)D_4\left(1 - \frac{D_4}{P_4}\right) + (h_5 + d_5\theta_5)D_5\left(1 - \frac{D_5}{P_5}\right)\end{array}\right\}}}$$

For optimum value, we have to show that $\frac{d^2\tilde{T}_{cg}}{dT^2} > 0$.

Now

$$\frac{d^2\tilde{T}_{cg}}{dT^2} = \frac{1}{12}\left[\frac{d^2\tilde{T}_{cg1}}{dT^2} + 3\frac{d^2\tilde{T}_{cg2}}{dT^2} + 4\frac{d^2\tilde{T}_{cg3}}{dT^2} + 3\frac{d^2\tilde{T}_{cg4}}{dT^2} + \frac{d^2\tilde{T}_{cg5}}{dT^2}\right]$$

$$= \frac{1}{6T^3}[S_1 + 3S_2 + 4S_3 + 3S_4 + S_5]$$

Take $\frac{D_1}{P_1} < 1, \frac{D_2}{P_2} < 1, \frac{D_3}{P_3} < 1, \frac{D_4}{P_4} < 1$ and $\frac{D_5}{P_5} < 1$, then in each case, T exists . Then for each value of T we can see $\frac{d^2\tilde{T}_{cg}}{dT^2} > 0$. Hence we got defuzzified value of total cost function in graded mean integration representation (GMIR) method i.e. $d\tilde{T}_{cg}$ is minimum.

6. Numerical Example:

First, we represent the case of vague value as the type of pentagonal fuzzy number. Consider the inventory system with the following parametric values. Suppose $\tilde{S}_c = (50, 52, 54, 56, 58)$, $\tilde{h}_c = (6, 7, 8, 9, 10)$, $\tilde{\theta} = (0.006, 0.008, 0.010, 0.012, 0.014)$, $\tilde{P}_r = (500, 520, 540, 560, 580)$, $\tilde{D} = (450, 470, 490, 510, 530)$, $\tilde{d} = (1.1, 1.2, 1.3, 1.4, 1.5)$ are all pentagonal fuzzy number and We solve this by using Matlab R2011b software. The solution of fuzzy total cost $\tilde{T}_{cg} = 198.1923$ with cycle time T = 0.5449

7. Sensitivity analysis :

A sensitivity analysis is carried out to study the effect of changes in parameters $\tilde{S}_c$, $\tilde{h}_c$, $\tilde{\theta}$, $\tilde{P}_r$, $\tilde{D}$ and $\tilde{d}_c$. We use Matlab R2011b software for calculation of the total cost in different defuzzification methods and plotting the graphs. In this analysis we change the value of a specific parameter keeping all other parameter remains constant.

Table 1. Sensitivity analysis for $\tilde{S}_c$

$\tilde{S}_c$	Time (Yrs)	Total Cost
(46, 48, 50, 52, 54)	0.5244	190.7106
(48, 50, 52, 54, 56)	0.5347	194.4874
(50, 52, 54, 56, 58)	0.5449	198.1923
(55, 57, 59, 61, 63)	0.5696	207.1647
(60, 62, 64, 66, 68)	0.5932	215.7644

Table 2. Sensitivity analysis for $\tilde{h}_c$

$\tilde{h}_c$	Time (Yrs)	Total Cost
(4, 5, 6, 7, 8)	0.6290	171.7026
(5, 6, 7, 8, 9)	0.5825	185.4211
(6, 7, 8, 9, 10)	0.5449	198.1923
(7, 8, 9, 10, 11)	0.5138	210.1889
(8, 9, 10, 11, 12)	0.4875	221.5368

Table 3. Sensitivity analysis for $\tilde{\theta}$

$\tilde{\theta}$	Time (Yrs)	Total Cost
(0.002, 0.004, 0.006, 0.008, 0.01)	0.5451	198.1280
(0.004, 0.006, 0.008, 0.01, 0.012)	0.5450	198.1601
(0.006, 0.008, 0.01, 0.012, 0.014)	0.5449	198.1923
(0.008, 0.01, 0.012, 0.014, 0.016)	0.5448	198.2244
(0.01, 0.012, 0.014, 0.016, 0.018)	0.5447	198.2565

Table 4. Sensitivity analysis for $\tilde{d}_c$

$\tilde{d}_c$	Time (Yrs)	Total Cost
(0.9, 1.0, 1.1, 1.2, 1.3)	0.5450	198.1675
(1.0, 1.1, 1.2, 1.3, 1.4)	0.5450	198.1799
(1.1, 1.2, 1.3, 1.4, 1.5)	0.5449	198.1923
(1.2, 1.3, 1.4, 1.5, 1.6)	0.5449	198.2046
(1.3, 1.4, 1.5, 1.6, 1.7)	0.5449	198.2107

Table 5. Sensitivity analysis for $\tilde{P}_r$

$\tilde{P}_r$	Time (Yrs)	Total Cost
(460, 480, 500, 520, 540)	1.1727	92.0972
(480, 500, 520, 540, 560)	0.6904	156.4317
(500, 520, 540, 560, 580)	0.5449	198.1923
(520, 540, 560, 580, 600)	0.4690	230.2952
(540, 560, 580, 600, 620)	0.4209	256.5242

Table 6. Sensitivity analysis for $\widetilde{D}$

$\widetilde{D}$	Time (Yrs)	Total Cost
(430, 450, 470, 490, 510)	0.4702	229.6892
(440, 460, 480, 500, 520)	0.5026	214.8914
(450, 470, 490, 510, 530)	0.5449	198.1923
(460, 480, 500, 520, 540)	0.6031	179.0607
(480, 500, 520, 540, 560)	0.6896	131.5267

Fig. 3. Sensitivity analysis for Set-up cost

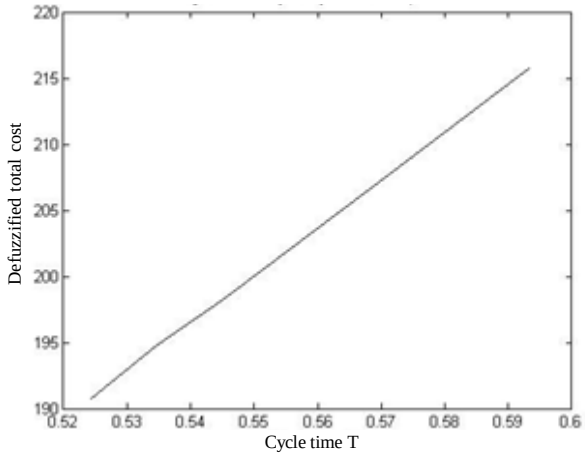


Fig. 4. Sensitivity analysis for Holding cost

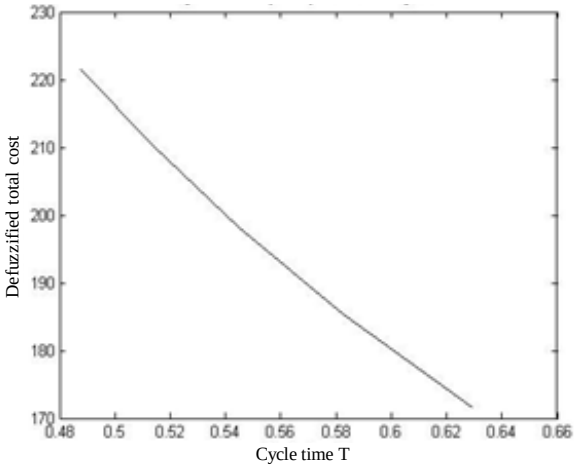


Fig. 5. Sensitivity analysis for Deterioration rate

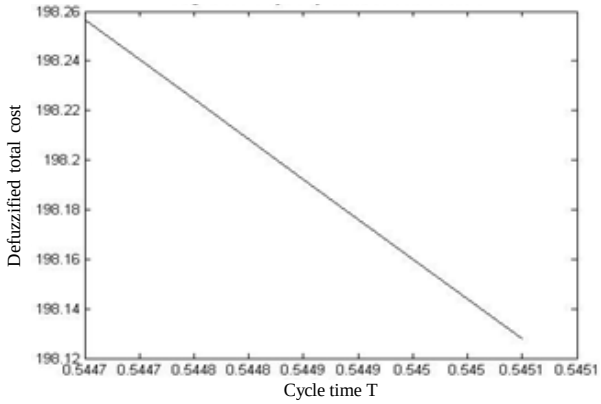


Fig. 6. Sensitivity analysis for Deterioration cost

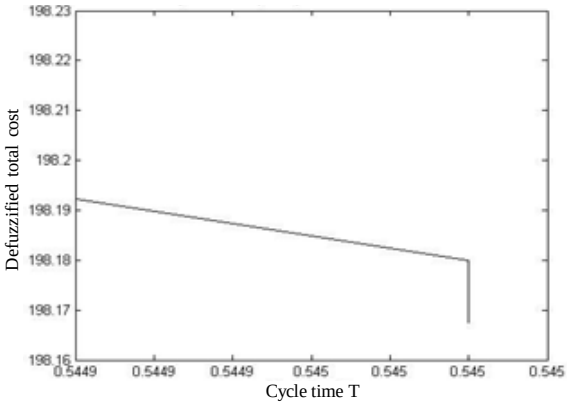


Fig. 7. Sensitivity analysis for Production rate

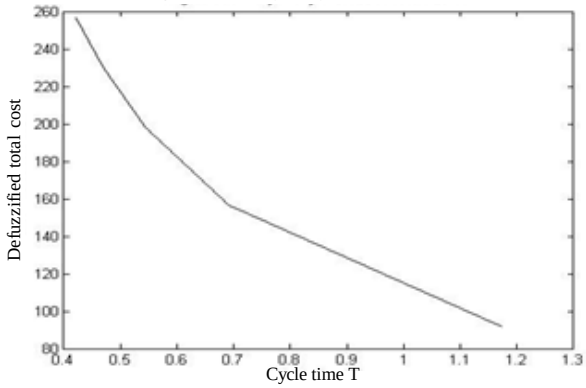
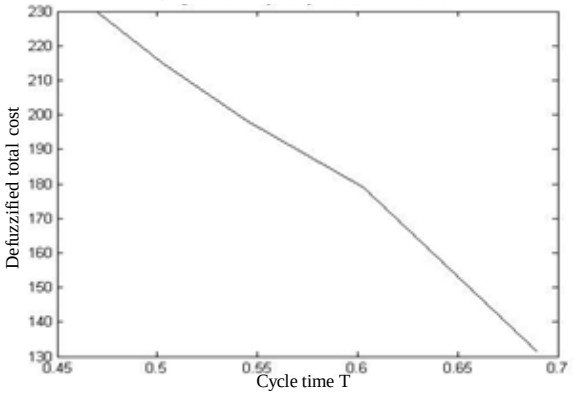


Fig. 8. Sensitivity analysis for Demand rate



All the above observations can be sum up as follows:

i) In table 1 & fig. 3 Shows that, with increase of the value of the parameter of $\tilde{S}_c$, keeping all other parameters unchanged , the total cost $\tilde{T}_{cg}$ increases with the increase of cycle time T.

ii) In table 2, fig. 4, Shows that, with increase of the value of the parameter of $\tilde{h}_c$, keeping all other parameters unchanged, the total cost $\tilde{T}_{cg}$ increases with the decrease of cycle time T.

iii) In table 3, fig.5, Shows that, with increase of the value of the parameter of $\tilde{\theta}$, keeping all other parameters unchanged , the total cost $\tilde{T}_{cg}$ increases slowly with the decrease of cycle time T.

iv) In table 4, fig. 6, Shows that, with increase of the value of the parameter of $\tilde{d}_c$, keeping all other parameters unchanged, the total cost $\tilde{T}_{cg}$ increases slowly with the decrease of cycle time T.

v) In table 5, fig. 7, Shows that, with increase of the value of the parameter of $\tilde{P}_r$, keeping all other parameters unchanged, the total cost $\tilde{T}_{cg}$ increases with the decrease of cycle time T.

vi) In table 6, fig. 8, Shows that, with increase of the value of the parameter of $\tilde{D}$, keeping all other parameters unchanged, the total cost $\tilde{T}_{cg}$ decreases with the increase of cycle time T.

8. Conclusion and Scope :

In this paper we developed a production fuzzy inventory model of deteriorating items. This model has been developed for single item without shortages. The set-up cost, holding cost, deterioration cost, deteriorating rate, production rate and demand are represented by pentagonal fuzzy numbers. The optimum result of fuzzy model is defuzzified by using graded mean integration representation method. Numerical example is also provided to illustrate the proposed model. As per the Sensitivity analysis the observation narrated in section 7. Due to fuzzy model, we did not get any absurd value. So the decision maker can plan for apply this model to get the optimum value of total cost, and for other related parameters, after analyzing the result. The problem is open. The result can be compared by taking different fuzzy number and defuzzifying in other methods. This can be further developed for different demand also.

9. Acknowledgement

The first author would like to acknowledge the Department of Science & Technology, Govt. of India for providing INSPIRE fellowship.

References

1. Chang, S.C., 'Fuzzy production inventory for fuzzy product quality with triangular fuzzy number', *Fuzzy set and Systems*, Vol. 107, no. 1, pp. 37-57. DOI: 10.1016/S0165-0114(97)00350-3 (1999).
2. Covert, R.P., Philip, G.C., 'An EOQ model for items with Weibull distribution deterioration', *AIIE Transactions*, Vol. *5*, pp. 323-326 (1973).
3. De, P.K., Rawat, A., 'A fuzzy inventory model without shortages using triangular fuzzy number", *Fuzzy Information and Engineering*, Vol. 1, pp. 59-68 (2011).
4. Dubois, D., Prade. H., 'Operations on Fuzzy numbers', *Int. J. of Systems Sc.*, Vo. *9* no. 6, pp. 613-626 (1978).
5. Dutta, D., Kumar, P., 'Fuzzy inventory model for deteriorating items with shortages under fully backlogged

condition', *Int. J. of Soft Computing & Engg.*, Vol. 3, no. 2, pp. 393-398 (2013).

6. Gen, M., Tsujimura, Y., Zheng D., 'An application of fuzzy set theory to inventory control models', *Computers and Industiral Engineering*, Vol. *33*, no. 3, pp.553-556. DOI: 10.1016/S0360-8352(97)00191-5 (1997).
7. Ghare, P.M., Schrader, S.F., 'A model for exponentially decaying inventory', *Journal of Industrial Engineering*, Vol. *14*, pp. 238-243 (1963).
8. Harris, F., ' Operations and Cost', *Factory Management Series*, AW Shaw Co. Englewood Cliffs, NJ, pp. 48-52 (1915).
9. Indrajitsingha, S.K., Routray, S. cS., Paikray, S.K., Misra U., c 'Fuzzy economic production quantity model with time dependent demand rate', *LogForum*, Vol. 12, no. 3, pp. 193-198. DOI: 10.17270/J.LOG.2016.3.1 (2016).
10. Indrajitsingha, S.K., Samanta, P.N., Misra, U.K., 'An economic production quantity model with time dependent demand under fuzzy environment', *Int. J. of Modern Sc. & Engg. Tech.*, Vol. *2*, no.12, pp. 1-8 (2015).
11. Indrajitsingha, S.K., Samanta, P.N., Misra, U.K., 'Fuzzy inventory model with shortages under fully backlogged using Signed distance method', *Int. J. Res. in Appl. Sc & Engg. Tech.*, Vol. 4, no. 1, pp. 197-203 (2016).
12. Jaggi, C.K., Pareek, S., Sharma, A., Nidhi, 'Fuzzy inventory model for deteriorating items with time-varying demand and shortages', *American Journal of Operational Research*, Vol. 2 no. 6, pp. 81-92, DOI: 10.5923/ j.ajor.20120206.01 (2012).
13. Kacpryzk, J., Staniewski, P., 'Long-term inventory policy-making through fuzzy-decision making models', *Fuzzy Sets and Systems, Vol.* 8, pp. 117-132 (1982).
14. Kaufmann, A., Gupta, M., Medan, M., 'Introduction to Fuzzy Arithmetic, Theory and Applications', *Van Nostrand Reinhold Company*, New York (1991).
15. Kumar, S., Rajput, U.S., 'Fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging', *Applied Mathematics*, Vol. *6*, pp. 496-509, http://dx.doi.org/10.4236/am.2015.63047 (2015).
16. Maragatham, M., Lakshmidevi, P.K., 'A fuzzy inventory model for deteriorating items with price dependent demand', *Int. J. of Fuzzy Mathematical Archive*,Vol. *5* no. 1, pp. 39-47 (2014).
17. Mishra, S., Barik, S., Paikray, S.K., Misra, U.K., Indrajitsingha S.K., 'An inventory control model of deteriorating items in fuzzy environment', *Global J. of Pure & Applied Mathematics*, Vol. *11* no. 3, pp. 1301-1312 (2015).
18. Nagar, H., Surana, P., 'Fuzzy inventory model for deteriorating items with fluctuating demand and using inventory parameters as pentagonal fuzzy numbers', *J. of Comp. & Math. Sc.*, Vol. *6* no. 2, pp. 55-66 (2015).
19. Park, K.S., 'Fuzzy-set theoretic interpretation of economic order quantity', *IEEE Transactions on systems, Man, and Cybernatics SMC-17*, pp. 1082-1084 (1987).
20. Ranganathan, V., Thirunavukarasu, P., 'An inventory control model for constant deterioration in fuzzy environment', *International Journal of Fuzzy Mathematics and Systems*, Vol. *4* no. 1, pp. 17-26 (2014).
21. Ranganathan, V., Thirunavukarasu, P., 'Fuzzy inventory model under immediate return for deficient items', *International Journal of Mathematical Analysis*, pp.307-316 (2015).
22. Raula, P., Indrajitsingha, S.K., Samanta, P.N., Misra, U.K., 'A fuzzy inventory model for constant deteriorating item by using GMIR method in which inventory parameters treated as HFN', *Open J. of Appl. & Theoretical Math.*, Vol. *2* no. 1, pp. 13-20 (2016).

23. Sahoo, A.K., Samanta, P.N., Indrajitsingha, S.K., Misra, U.K., 'Fuzzy inventory model with time dependent demand rate and without shortages using pentagonal fuzzy numbers', *J. of Global Economis, Management and Business Research*, Vol. *7* no. 4, pp. 313-318 (2016).
24. Sahoo, N.K., Mohanty, B.S., Tripathy, P.K., 'Fuzzy inventory model with exponential demand and time varying deterioration', *Global J. of Pure & Applied Mathematics*, Vol. *12* no. 3, pp. 2573-2589 (2016).
25. Sen, N., Nath, B.K., Saha, S., 'A fuzzy inventory model for deteriorating items based on different defuzzification techniques', *American J. of Math. & Stat.*, Vol. *6* no. 3, pp. 128-137, doi:10.5923/j.ajms.20160603.07 (2016).
26. Singh, C., Singh, S.R., 'Vendor-Buyers relationship model for deteriorating items with shortages fuzzy trapezoidal cost and inflation', *Yugoslav J. of Operations Research*, Vol. 23 no. 1, pp. 73-85, doi:10.2298/YJOR110421022S (2013).
27. Syed, J.K., Aziz L.A., 'Fuzzy inventory model without shortage using signed distance method', *Applied Mathematics & Informatics Sciences*, Vol. *1* no. 2, pp. 203-209 (2007).
28. Vujosevic, M., Petrovic, D., Petrovic, R., 'EOQ formula when inventory cost is fuzzy', *International Journal of Production economics*, Vol. *45* no. 1-3, pp. 499-504. DOI: 10.1016/0925-5273(95)00149-2 (1996).
29. Yao, J.S., Lee, H.M., 'Fuzzy inventory with backorder for fuzzy order quantity', *Information Sciences*, Vol. *93* no. 3-4, pp. 283-319. DOI: 10.1016/0020-0255(96)00074-6 (1996).
30. Zadeh, L.A., 'Fuzzy Sets', *Information Control*, Vol. *8* no. 3, pp. 338-353 (1965).
31. Zadeh, L.A., Bellman, R.E., 'Decision making in a fuzzy environment', *Management Science*, Vol. *17*, pp. 140-164 (1970).
32. Zimmermann, H.J., 'Fuzzy Set theory and its Applications', 3rd *Ed., Dordrecht: Kluwar, academic Publishers, Boston* (1996).