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Another Type of Weakly Closed sets on Semi α- Regular

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Abstract

In this paper we introduce and study the new class of closed sets called semi-α-regular weakly closed (briefly sαrw-closed) set in the topological spaces. This new class of set lies between the class of α-closed sets and the class of generalised semi closed sets. We study the fundamental properties of this class of sets.

Key words: Semi-closed set, α-closed sets, rw-closed, αrw-closed set, gs-closed set, sαrw -closed set and topological space.

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1. Introduction

In 1970 Levine[10], first introduced the concept of generalized closed (briefly g-closed) sets were defined and investigated. Regular open sets and rw-open sets have been introduced and investigated by Stone[16] and Benchalli[3] respectively. Levine[10,11], Sundaram and Sheik john[17] and many mathematicians have been, introduced and investigated semi open sets, generalized closed sets, regular semi open sets, ω-closed sets, semi generalized closed sets, weakly generalized closed sets, strongly generalized closed sets, generalized pre-regular closed sets, regular generalized closed sets, and generalized α-generalized closed sets respectively. Maki *et.al*[12,13] introduced and studied generalized α-closed sets and α-generalized closed sets. S.S. Benchalli *et.al.*[3] studied ωα -closed sets in topological spaces. Recently, R. S. Wali and Mendalgeri[20,22] introduced and studied the concepts of α-regular w-closed (briefly αrw-closed) sets in topological spaces. In this paper we define new generalization of closed set called Semi α regular weakly closed (briefly sαrw-closed) set which lies between α-closed set and gs-closed set. Also we study their fundamental properties.

2. Preliminaries :

Definition 2.1 : A subset A of X, is called **semi-open** set[11] if $A \subseteq cl(int(A))$ and semi-closed set if int(cl(A)) ⊆ A.

Definition 2.2 : A subset A of X, is called pre-**open** set[14] if A ⊆ int (cl(A)) and pre-closed set if cl(int(A)) ⊆ A.

Definition 2.3 : A subset A of X, is called **α-open** set[9] if A ⊆ int (cl(int(A))) and α -closed set if cl(int(cl(A)))⊆ A.

Definition 2.4: A subset A of X, is called **semi-pre open** set[1] (β-open[1] if A⊆cl (int(cl(A)))) and a semi-pre closed set (β-closed) if int(cl(int(A)))⊆A.

Definition 2.5 : A subset A of X, is called **regular open** set[16] if A = int(clA)) and a regular closed set if A = cl(int(A)).

Definition 2.6: A subset A of X, is called **Regular α-open** set[18] (briefly, rα-open) if there is a regular open set U s.t U ⊆ A ⊆ αcl(U).

Definition 2.7: A subset A of X, is called **Regular semi open** set[5] if there is a regular open set U such that U ⊆ A ⊆ cl(U).

Definition 2.8: A subset A of a topological space (X, τ) is called

1. α-generalized closed set(briefly α g -closed)[12] if α-cl(A)⊆U whenever A⊆U and U is open in X.
2. Regular generalized closed set(briefly rg-closed)[15] if cl(A)⊆U whenever A⊆U and U is regular open in X.
3. Generalized semi-pre closed set(briefly gsp-closed)[7] if spcl(A) ⊆ U whenever A⊆U and U is open in X.
4. ωα- closed set[3] if α-cl(A) ⊆ U whenever A⊆ U and U is ω-open in X.
5. Generalized ωα-closed (briefly gωα-closed) set[3] if α-cl(A⊆ U whenever A⊆U and U is ωα -open in X.
6. Generalized regular closed (briefl gr –closed) set[4] if rcl(A)⊆ U whenever A⊆ U and U is open in X.
7. α -regular weakly closed (briefly αrw-closed)set[20,22] if α-Cl(A)⊆U whenever A ⊆ U and U is rω-open in X

The complement of the above mentioned closed sets are their open sets.

3. weakly-closed sets in terms of Semi α -regular :

Definition 3.1: A subset A of X is called a semi α- regular weakly closed set (briefly sαrw-closed set) if Scl(A)⊆U whenever A ⊆ U and U is αrw-open in X. We denote the collection of all sαrw-closed sets in X by sαrw (X).

Theorem 3.2: Every α-closed set in X is sαrw-closed set but converse is not true

Proof: Let A be α-closed set in X. Let U be any -open set in X s.t A ⊆ U. Since A is α-closed, we have Scl(A) = A⊆ U, and Scl(A)⊆U. Hence A is sαrω-closed set in X.

Example 3.3: Let X= {a, b, c, d} and τ={X, ϕ,{a},{b, c},{a, b, c}} then the set A={b, d} is sαrω-closed set but not α-closed in X.

Theorem 3.4: Every sαrw-closed set is gs-closed set in X but converse is not true .

Proof: Let A be sαrw -closed set in X. Let U be any open set in X s.t. A ⊆ U. Since every open set is rω-open set[5] and A is sαrw-closed set, we have SCl(A)⊆U, SCl(A) ⊆U, U is open in X. Hence A is gs-closed set in X.

Example 3.5: Let X={a, b, c,d} and τ ={X, ϕ,{a},{b, c} {a, b, c}} then the set A={ b} is gs closed set but not sαrw -closed set in X.

Remark 3.6: The Union of two sαrw - closed subsets of X need not be sαrw -closed set in X.

Example 3.7: Let X = {a, b, c, d} be with topology τ = {X, φ, {a}, {b, c},{a, b, c}}. Let A = {a} and B = {b, c} be two sαrw -closed subsets of X. But AUB = {a, b, c} which is not contained in sαrw -closed set in X. Hence union of two sαrw -closed sets is not sαrw -closed set in X.

Remark 3.8: The intersection of two sαrw -closed sets in X is generally not an sαrw -closed set in X.

Example 3.9: Let X = {a, b, c, d} be with topology τ = { X, φ, {a}, {b, c},{a, b, c}}. Let A = {b, c} and B ={b, d} be two sαrw -closed subsets of X. But $A \cap B$ = {b} which is not contained in sαrw -closed set in X. Hence intersection of two sαrw -closed sets is not sαrw -closed set in X.

Theorem 3.10: If a subset A of topological space X is an sαrw -closed set in X then SCl(A) - A does not contain any non empty rω -closed set in X.

Proof: Let A is an sαrw-closed set in X and suppose F be an non empty rω -closed subset of SCl(A) – A. $F \subseteq$ SCl(A) – A => $F \subseteq$ SCl(A)) $\cap$ (X– A) => $F \subseteq$ SCl(A)--(1) & $F \subseteq$ X–A => $A \subseteq$ X– F and X – F is rω-open set and A is an sαrw-closed set, SCl(A) $\subseteq$ X– F=> $F \subseteq$ X – SCl(A) -- (2) from equations (1) and (2) we get $F \subseteq$ SCl(A)$\cap$(X – SCl(A)) = ϕ=> F = ϕ. Thus SCl(A) – A does not contain any non empty rω closed set in X. However the converse of the above theorem need not be true as seen from the following example.

Example 3.11: Let X={a, b, c, d} and τ = {X, ϕ,{a},{b ,c },{a, b, c}} then the set A={b},Scl(A) – A = {b, c} – {b} ={c} does not contain non empty rω closed set in X, but A is not sαrw-closed set in X.

4. Characterisation of is sαrw -closed set :

Theorem 4.1: If A is regular open and αgr-closed then A is sαrw -closed set in X.

Proof: Let A be regular open and αgr-closed in X, Let U be any rω-open set in X s.t. $A \subseteq U$ Since A is regular open and αgr-closed in X, by definition, SCl(A)$\subseteq$A then SCl(A)$\subseteq A \subseteq$U Hence A is sαrw -closed set in X.

Remark 4.2: If A is both regular open and sαrw -closed, then A need not be αgr -closed in general as seen from the following example.

Example 4.3: Let X={a, b, c, d} with the topology τ = {X,,{a},{b, c}, {a, b, c}}. Let A={b,c} is both regular open and sαrw -closed but not αgr closed in X.

Theorem 4.4: If A is ω-open and $\omega\alpha$-closed then A is sαrw -closed set in X.

Proof: Let A be ω-open and $\omega\alpha$-closed in X, Let U be any rω-open set in X s.t. $A \subseteq U$. Since A is ω-open and $\omega\alpha$-closed in X, by definition, SCl(A)$\subseteq$A then SCl(A)$\subseteq A \subseteq$U. Hence A is sαrw -closed set in X.

Remark 4.5: If A is both ω-open and sαrw -closed, then A need not be $\omega\alpha$- closed in general, as seen from the following example.

Example 4.6: Let X={a, b, c, d} with the topology τ = {X,ϕ,{a},{b, c}, {a, b, c}}. Let A={a} is both ω-open and sαrw-closed but not $\omega\alpha$ -closed in X.

Theorem 4.7: If A is open and αg-closed then A is sαrw -closed set in X.

Proof: Let A be open and αg-closed in X, Let U be any rω-open set in X s.t. $A \subseteq U$. Since A is open and αg-closed in X, by definition, SCl(A) $\subseteq$A then SCl(A) $\subseteq A \subseteq$U. Hence A is sαrw -closed set in X.

Remark 4.8: If A is both open and sαrw -closed, then A need not be αg- closed in general, as seen from the following example.

Example 4.9: Let X={a, b, c, d} with the topology τ = {X,ϕ{a},{b, c}, {a, b, c}}. Let A={b, c} is both open and sαrw closed but not αg -closed in X.

Theorem 4.10: If A is regular open and rwg-closed then A is sαrw -closed set in X.

Proof: Let A be regular open and rwg-closed in X, Let U be any rω-open set in X s.t. $A \subseteq U$ Since A is regular open and rwg-closed in X, by definition, Cl(int(A))$\subseteq$A then we know that Cl(int(A)) $\subseteq$ Cl(int(cl(A))) $\subseteq A \subseteq$U Hence SCl(A) $\subseteq$U.

Hence A is sαrw -closed set in X.

Remark 4.11: If A is both regular open and sαrw -closed, then A need not be rwg – closed in general, as seen from the following example.

Example 4.12: Let X={a, b, c, d} with the topology τ = {X,ϕ,{a},{b, c}, {a, b, c}}. Let A={a} is both regular open and sαrw -closed but not rwg -closed in X

Remark 4.13: If A is both open and sαrw -closed, then A need not be wg - closed in general, as seen from the following example.

Example 4.14: Let X={a, b, c, d} with the topology τ = {X,ϕ,{a},{b, c},{a, b, c}}. Let A={b,c} is both open and sαrw closed but not wg -closed in X.

Remark 4.15: If A is both open and sαrw -closed, then A need not be gp - closed in general, as seen from the following example.

Example 4.16: Let X={a, b, c, d} with the topology τ = {X,ϕ,{a},{b, c}, {a, b, c}}. Let A={b, c} is both open and sαrw-closed but not gp -closed in X.

Theorem 4.17: If a subset A of a topological space X in both regular open and sαrw-closed then it is α-closed.

Proof: Suppose a subset A of a topological space X is regular open and sαrw-closed, As every regular open is rω-open. Now A⊆ A then definition of sαrw-closed,
SCl(A) ⊆ A and also A⊆ SCl(A) then SCl(A)=A. Hence A is α-closed.

Theorem 4.18: If a subset A of a topological space X is both regular semi open and gprw-closed then it is sαrw-closed.

Proof: Let A be an regular semi open and gprw-closed set in X. Let A⊆ U and U be rω-open in X. Now A⊆A by hypothesis pcl(A) ⊆A then we know that pcl(A)⊆Scl(A) ⊆ A, hence Scl(A) ⊆ U therefore A is sαrw-closed set in X.

Remark 4.19: If A is both regular semi open and sαrw -closed, then A need not be gprw - closed in general, as seen from the following example.

Example 4.20: Let X={a, b, c, d} with the topology τ = {X,ϕ,{a},{b, c}, {a, b, c}}. Let A={b,c} is both regular semi open and sαrw-closed but not gprw -closed in X.

Theorem 4.21: If a subset A of a topological space X is both regular semi open and rgw-closed then it is sαrw-closed.

Proof: Let A be an regular semi open and rgw-closed set in X. Let A⊆ U and U be rω-open in X. Now A⊆ A by hypothesis cl(int(A)) ⊆ A then we know that
cl(int(A)) ⊆ cl(int(cl(A))) ⊆ A, hence Scl(A) ⊆ U therefore A is sαrw - closed set in X.

Remark 4.22: If A is both regular semi open and sαrw-closed, then A need not be rgw - closed in general, as seen from the following example.

Example 4.23: Let X={a, b, c, d} with the topology τ = {X,ϕ,{a},{b, c}, {a, b, c}}. Let A={b,c} is both regular semi open and sαrw -closed but not rgw -closed in X.

Conclusion

In the present work, a new class of sets called sαrw -Closed sets in Topological spaces is introduced and some of their properties are studied. This new class of sets widens the scope to do further research in the areas like Bitopological Spaces, Soft topological spaces and Fuzzy Topological Spaces.

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