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Multi-Objective Welded Beam Optimization using Neutrosophic Optimization Technique: A Comparative Study

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Abstract

This paper investigates multi–objective Neutrosophic Optimization (NSO) approach to optimize the cost of welding and deflection at the tip of a welded steel beam, while the maximum shear stress in the weld group, maximum bending stress in the beam, and buckling load of the beam have been considered as constraints. The problem of designing an optimal welded beam consists of dimensioning a welded steel beam and the welding length so as to minimize its cost, subject to the constraints as stated above. The purpose of the present study firstly to investigate the effect of truth, indeterminacy and falsity membership function in neutrosophic optimization in perspective of welded beam design and secondly is to analyse the results obtained by different optimization methods like fuzzy, intuitionistic fuzzy so that the welding cost of the welded steel beam become most cost effective with minimum deflection. Specifically based on truth, indeterminacy and falsity membership function, a multi objective NSO algorithm has been developed to optimize the welding cost, subjected to a set of constraints. It has been shown that NSO is an efficient method in finding out the optimum value in comparison to other iterative methods for nonlinear welded beam design in imprecise environment till investigated. Numerical example is also given to demonstrate the efficiency of the proposed NSO approach.

Key words: Neutrosophic Set, Single Valued Neutrosophic Set, Neutrosophic Optimization, Multi-Objective welded beam optimization.

Subject classification code:90C30,90C70,90C90

1. Introduction

Welding, a process of joining metallic parts with the application of heat or pressure or the both, with or without added material, is an economical and efficient method for obtaining permanent joints in the metallic parts. This welded joints are generally used as a substitute for riveted joint or can be used as an alternative method for casting or forging. The welding processes can broadly be classified into following two groups, the welding process that uses heat alone to join two metallic parts and the welding process that uses a combination of heat and pressure for joining (Bhandari. V. B). However, above all the design of welded beam should preferably be economical and durable one. Since decades, deterministic optimization has been widely used in practice for optimizing welded connection design. These include mathematical optimization algorithms (Ragsdell & Phillips 1976) such as APPROX (Griffith & Stewart's) successive linear approximation, DAVID (Davidon Fletcher Powell with a penalty function), SIMPLEX (Simplex method with a penalty function), and RANDOM (Richardson's random method) algorithms, GA-based methods[2,4,16,17] particle swarm optimization[3], harmony search method[5], and Big-Bang Big-Crunch (BB-BC) (O. Hasançebi, 2011) algorithm. SOPT[7], subset simulation (Li 2010), improved harmony search algorithm[8], were other methods used to solve this problem. Recently a robust and reliable ***H***" static output feedback (SOF) control for nonlinear systems[13] and for continuous-time nonlinear stochastic systems[13] with actuator fault in a descriptor system framework have been studied. All these deterministic optimizations aim to search the optimum solution under given constraints without consideration of uncertainties. So, while a deterministic optimization approach is unable to handle structural performances such as imprecise stresses and deflection etc. due to the presence of uncertainties, to get rid of such problem fuzzy[1], intuitionistic fuzzy[9], Neutrosophic[10] play great roles.

Traditionally structural design optimization is a well known concept and in many situations it is treated as single objective form, where the objective is known the weight or cost function. The extension of this is the optimization where one or more constraints are simultaneously satisfied next to the minimization of the weight or cost function. This does not always hold good in real world problems where multiple and conflicting objectives frequently exist. In this consequence a methodology known as multi-objective optimization (MOSO) is introduced

So to deal with different impreciseness such as stresses and deflection with multiple objective, we have been motivated to incorporate the concept of neutrosophic set in this problem, and have developed multi-objective neutrosophic optimization algorithm to optimize the optimum design.

Usually Intuitionistic fuzzy set, which is the generalization of fuzzy sets, considers both truth membership and falsity membership that can handle incomplete information excluding the indeterminate and inconsistent information while neutrosophic set can quantify indeterminacy explicitly by defining truth, indeterminacy and falsity membership function independently. Therefore, Wang *et.al.* (2010) presented such set as single valued neutrosophic set (SVNS) as it comprised of generalized classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set and Para-consistent set.

As application of SVNS optimization method is rare in welded beam design, hence it is used to minimize the cost of welding by considering shear stress, bending stress in the beam, the buckling load on the bar, the deflection of the beam as constraints. Therefore the result has been compared among three cited methods in each of which impreciseness has been considered completely in different way.

Moreover using above cited concept, a multi-objective neutrosophic optimization algorithm has been developed to optimize three bar truss design[14], and to optimize riser design problem[15].

However, the factors governing of former constraints are height and length of the welded beam, forces on the beam, moment of load about the centre of gravity of the weld group, polar moment of inertia of the weld group respectively. While, the second constraint considers forces on the beam, length and size of the weld, depth and width of the welded beam respectively. Third constraint includes height and width of the welded beam. Fourth constraints consists of height, length, depth and width of the welded beam. Lastly fifth constraint includes height of the welded beam. Besides, flexibility has been given in shear stress, bending stress and deflection only, hence all these parameters become imprecise in nature so that it can be considered as neutrosophic set to from truth, indeterminacy and falsity membership functions Ultimately, neutrosophic optimization technique has been applied on the basis of the cited membership functions and outcome of such process provides the minimum cost of welding ,minimum deflection for nonlinear welded beam design. The comparison of results shows difference between the optimum value when partially unknown information is fully considered or not.

2. Multi-Objective Structural Model

In sizing optimization problems, the aim is to minimize multi objective function, usually the cost of the structure, deflection under certain behavioural constraints which are displacement or stresses. The design variables are most frequently chosen to be dimensions of the height, length, depth and width of the structures. Due to fabrications limitations the design variables are not continuous but discrete for belongingness to a certain set. A discrete structural optimization problem can be formulated in the following form

$$Minimize\ C(X) \tag{1}$$

$$Minimize\ \delta(X)$$

$$subject\ to\ \sigma_i(X) \le [\sigma_i(X)], i = 1, 2, ..., m$$

$$X_j \in R^d, \quad j = 1, 2, ..., n$$

where $C(X)$, $\delta(X)$ and $\sigma_i(X)$ as represent cost function, deflection and the behavioural constraints respectively whereas $[\sigma_i(X)]$ denotes the maximum allowable value , '*m*' and '*n*' are the number of constraints and design variables respectively. A given set of discrete value is expressed by R^d and in this paper objective functions are taken as

$$C(X) = \sum_{t=1}^{T} c_t \prod_{n=1}^{m} x_n^{tn} \text{ and } \delta(X)$$

and constraint are chosen to be stress of structures as follows

$\sigma_i(A) \lesssim \sigma_i$ with allowable tolerance σ_i^0 for $i = 1, 2, ..., m$

Where c_t is the cost coefficient of tth term and x_n is the n^{th} design variable respectively, m is the number of structural element, σ_i and σ_i^0 are the i^{th} stress, allowable stress respectively.

3. Mathematical preliminaries

3.1. Fuzzy Set

Let X be a fixed set. A fuzzy set $\tilde{A}$ of X is an object having the form $\tilde{A} = \{(x, T_{\tilde{A}}(x)) : x \in X\}$ where the function $T_{\tilde{A}} : X \to [0,1]$ defined the truth membership of the element $x \in X$ to the set $\tilde{A}$.

3.2. Intuitionistic Fuzzy Set

Let a set X be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}^i$ in X is an object of the form $\tilde{A}^i = \{< X, T_{\tilde{A}^i}(x), F_{\tilde{A}^i}(x) > | x \in X\}$ where $T_{\tilde{A}^i} : X \to [0,1]$ and $F_{\tilde{A}^i} : X \to [0,1]$ define the truth membership and falsity membership respectively, for every element of $x \in X,\ 0 \le T_{\tilde{A}^i}(x) + F_{\tilde{A}^i}(x) \le 1$.

3.3. Neutrosophic Set

Let a set be a space of points (objects) and $x \in X$. A neutrosophic set $\tilde{A}^n$ in X is defined by a truth membership function $T_{\tilde{A}^n}(x)$, an indeterminacy-membership function $I_{\tilde{A}^n}(x)$ and a falsity membership function $F_{\tilde{A}^n}(x)$ and having the form $\tilde{A}^n = \{< x, T_{\tilde{A}^n}(x), I_{\tilde{A}^n}(x), F_{\tilde{A}^n}(x) > | x \in X\}$. $T_{\tilde{A}^n}(x)$, $I_{\tilde{A}^n}(x)$ and $F_{\tilde{A}^n}(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is

$$T_{\tilde{A}^n}(x) : X \to]0^-, 1^+[$$
$$I_{\tilde{A}^n}(x) : X \to]0^-, 1^+[$$
$$F_{\tilde{A}^n}(x) : X \to]0^-, 1^+[$$

There is no restriction on the sum of $T_{\tilde{A}^n}(x)$, $I_{\tilde{A}^n}(x)$ and $F_{\tilde{A}^n}(x)$ so

$$0^- \le \sup T_{\tilde{A}^n}(x) + \sup I_{\tilde{A}^n}(x) + \sup F_{\tilde{A}^n}(x) \le 3^+.$$

3.4. Single Valued Neutrosophic Set

Let a set X be the universe of discourse. A single valued neutrosophic set $\tilde{A}^n$ over X is an object having the form $\tilde{A}^n = \{< x, T_{\tilde{A}^n}(x), I_{\tilde{A}^n}(x), F_{\tilde{A}^n}(x) > | x \in X\}$ where $T_{\tilde{A}^n} : X \to [0,1], I_{\tilde{A}^n} : X \to [0,1]$ and $F_{\tilde{A}^n} : X \to [0,1]$ are truth, indeterminacy and falsity membership functions respectively such that $0 \le T_{\tilde{A}^n}(x) + I_{\tilde{A}^n}(x) + F_{\tilde{A}^n}(x) \le 3$ for all $x \in X$.

3.5. Complement of Neutrosophic Set

Complement of a single valued neutrosophic set $\tilde{A}^n$ is denoted by $C(\tilde{A}^n)$ and its truth, indeterminacy and falsity membership functions are denoted by $T_{C(\tilde{A}^n)} : X \to [0,1], I_{C(\tilde{A}^n)} : X \to [0,1]$ and

$F_{C(\tilde{A}^n)} : X \to [0,1]$ where $T_{C(\tilde{A}^n)}(x) = F_{\tilde{A}^n}(x),\ I_{C(\tilde{A}^n)}(x) = 1 - F_{\tilde{A}^n}(x),\ F_{C(\tilde{A}^n)}(x) = T_{\tilde{A}^n}(x)$

3.6 Union of Neutrosophic Sets

The union of two single valued neutrosophic sets $\tilde{A}^n$ and $\tilde{B}^n$ is a single valued neutrosophic set $\tilde{U}^n$ denoted by

$$\tilde{U}^n = \tilde{A}^n \bigcup \tilde{B}^n = \left\{ \left(x, T_{\tilde{U}^n}(x), I_{\tilde{U}^n}(x), F_{\tilde{U}^n}(x) \right) \middle| x \in X \right\}$$

and is defined by the following conditions

(i) $T_{\tilde{U}^n}(x) = \max\left(T_{\tilde{A}^n}(x), T_{\tilde{B}^n}(x) \right),$

(ii) $I_{\tilde{U}^n}(x) = \max\left(I_{\tilde{A}^n}(x), I_{\tilde{B}^n}(x) \right),$

(iii) $F_{\tilde{U}^n}(x) = \min\left(F_{\tilde{A}^n}(x), F_{\tilde{B}^n}(x) \right)$ for all $x \in X$ for Type-I

Or in another way by defining following conditions

(i) $T_{\tilde{U}^n}(x) = \max\left(T_{\tilde{A}^n}(x), T_{\tilde{B}^n}(x) \right),$

(ii) $I_{\tilde{U}^n}(x) = \min\left(I_{\tilde{A}^n}(x), I_{\tilde{B}^n}(x) \right)$

(iii) $F_{\tilde{U}^n}(x) = \min\left(F_{\tilde{A}^n}(x), F_{\tilde{B}^n}(x) \right)$ for all $x \in X$ for Type-II

where $T_{\tilde{U}^n}(x),\ I_{\tilde{U}^n}(x),\ F_{\tilde{U}^n}(x)$ represent truth membership, indeterminacy-membership and falsity-membership functions of union of neutrosophic sets

3.7. Intersection of Neutrosophic Sets

The intersection of two single valued neutrosophic sets $\tilde{A}^n$ and $\tilde{B}^n$ is a single valued neutrosophic set $\tilde{E}^n$ is denoted by

$$\tilde{E} = \tilde{A}^n \bigcap \tilde{B}^n = \left\{ \left(x, T_{\tilde{E}^n}(x), I_{\tilde{E}^n}(x), F_{\tilde{E}^n}(x) \right) \middle| x \in X \right\}$$

and is defined by the following conditions

(i) $T_{\tilde{E}^n}(x) = \min\left(T_{\tilde{A}^n}(x), T_{\tilde{B}^n}(x) \right),$

(ii) $I_{\tilde{E}^n}(x) = \min\left(I_{\tilde{A}^n}(x), I_{\tilde{B}^n}(x) \right),$

(iii) $F_{\tilde{E}^n}(x) = \max\left(F_{\tilde{A}^n}(x), F_{\tilde{B}^n}(x) \right)$ for all $x \in X$ for Type-I

Or in another way by defining following conditions

(i) $T_{\tilde{E}^n}(x) = \min\left(T_{\tilde{A}^n}(x), T_{\tilde{B}^n}(x) \right),$

(ii) $I_{\tilde{E}^n}(x) = \max\left(I_{\tilde{A}^n}(x), I_{\tilde{B}^n}(x)\right)$

(iii) $F_{\tilde{E}^n}(x) = \max\left(F_{\tilde{A}^n}(x), F_{\tilde{B}^n}(x)\right)$ for all $x \in X$ for Type-II

where $T_{\tilde{E}^n}(x),\ I_{\tilde{E}^n}(x),\ F_{\tilde{E}^n}(x)$ represent truth membership, indeterminacy-membership and falsity-membership functions of union of neutrosophic sets

4. Mathematical Analysis

4.1. Neutrosophic Optimization Technique to Solve Minimization Type Multi-Objective Nonlinear Programming Problem

Decision making is nothing but a process of solving the problem that achieves goals under constraints. The outcome of the problem is a decision which should in an action. Decision making plays an important role in different subject such as field of economic and business, management sciences, engineering and manufacturing, social and political science, biology and medicine, military, computer science etc. It faces difficulty in prgress due to factors like incomplete and imprecise information which often present in real life situations. In the decision making process, the decision maker's main target is to find the value from the selected set with the highest degree of membership in the decision set and these values support the goals under constraints only. But there may be situations arise where some values from selected set cannot support, rather such values strongly against the goals under constraints which are non-admissible. In this case we find such values from the selected set with last degree of non-membership in the decision sets. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly belief in system. In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership, falsity-membership are independent to each other. So it is natural to adopt for that purpose the value from the selected set with highest degree of truth-membership, highest degree or leasts degree of indeterminacy-membership and least degree of falsity-membership on the decision set. These factors indicate that a decision making process takes place in neutrosophic environment.

Computational algorithm

Step-1: Solve the MONLP problem (2) as a single objective non-linear problem p times for each problem by taking one of the objectives at a time and ignoring the others. These solution are known as ideal solutions. Let x^k be the respective optimal solution for the k^{th} different objective and evaluate each objective values for all these k^{th} optimal solution.

Step-2: From the result of step-1, determine the corresponding values for every objective for each derived solution, pay-off matrix can be formulated as follows

$$\begin{bmatrix} f_1^*\left(x^1\right) & f_2\left(x^1\right) & \ldots\ldots & f_p\left(x^1\right) \\ f_1\left(x^2\right) & f_2^*\left(x^2\right) & \ldots\ldots\ldots & f_p\left(x^2\right) \\ \ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots \\ f_1\left(x^p\right) & f_2\left(x^p\right) & \ldots\ldots\ldots & f_p^*\left(x^p\right) \end{bmatrix}$$

Step-3: For each objective $f_k(x)$ find lower bound L_k^μ and the upper bound U_k^μ

$$U_k^T = \max\left\{ f_k\left(x^{r^*}\right)\right\} \quad and$$
$$L_k^T = \min\left\{ f_k\left(x^{r^*}\right)\right\} \quad where\ r = 1,2,....,k$$

For truth membership of objectives.

Step-4: We represent upper and lower bounds for indeterminacy and falsity membership of objectives as follows :

$$for\ k = 1,2,......p$$
$$U_k^F = U_k^T \quad and \quad L_k^F = L_k^T + t\left(U_k^T - L_k^T\right);$$
$$L_k^I = L_k^T \quad and \quad U_k^I = L_k^T + s\left(U_k^T - L_k^T\right)$$

Here t, s are predetermined real numbers in $(0,1)$

Step-5: Define truth membership, indeterminacy membership and falsity membership functions as follows

$for\ k = 1,2,......., p$

$$T_{f_k(x)}\left(f_k(x)\right) = \begin{cases} 1 & if \quad f_k(x) \le L^T_{f_k(x)} \\ 1 - \exp\left\{-\psi\left(\dfrac{U^T_{f_k(x)} - f_k(x)}{U^T_{f_k(x)} - L^T_{f_k(x)}}\right)\right\} & if \quad L^T_{f_k(x)} \le f_k(x) \le U^T_{f_k(x)} \\ 0 & if \quad f(x) \ge U^T_{f_k(x)} \end{cases}$$

$$I_{f_k(x)}\left(f_k(x)\right) = \begin{cases} 1 & if \quad f_k(x) \le L^I_{f(x)} \\ \exp\left\{\dfrac{U^I_{f_k(x)} - f_k(x)}{U^I_{f_k(x)} - L^I_{f_k(x)}}\right\} & if \quad L^I_{f_k(x)} \le f_k(x) \le U^I_{f_k(x)} \\ 0 & if \quad f_k(x) \ge U^I_{f_k(x)} \end{cases}$$

$$F_{f(x)}\left(f(x)\right) = \begin{cases} 0 & if \quad f(x) \le L^F_{f(x)} \\ \dfrac{1}{2} + \dfrac{1}{2}\tanh\left\{\left(f(x) - \dfrac{U^F_{f(x)} + L^F_{f(x)}}{2}\right)\tau_{f(x)}\right\} & if \quad L^F_{f(x)} \le f(x) \le U^F_{f(x)} \\ 1 & if \quad f(x) \ge U^F_{f(x)} \end{cases}$$

Step-6: Now neutrosophic optimization method for MONLP problem gives a equivalent nonlinear programming problem as:

$$Maximize\ \left(\alpha-\beta+\gamma\right) \tag{2}$$

$$such\ that$$

$$T_k\left(f_k\left(x\right)\right)\geq\alpha;\ I_k\left(f_k\left(x\right)\right)\geq\gamma;\ F_k\left(f_k\left(x\right)\right)\leq\beta;$$

$$\alpha+\beta+\upsilon\leq 3;\ \alpha\geq\beta;\alpha\geq\gamma;\ \alpha,\beta,\gamma\in\left[0,1\right];$$

$$g_j\left(x\right)\leq b_j\quad x\geq 0,$$

$$k=1,2,\ldots,p;\ j=1,2,\ldots,m$$

which is reduced to equivalent non linear programming problem as

$$Maximize\ \left(\theta-\eta+\kappa\right) \tag{3}$$

$$such\ that$$

$$f_k\left(x\right)+\frac{\theta\left(U_k^T-L_k^T\right)}{4}\leq L_k^T;$$

$$f_k\left(x\right)+\frac{\eta}{\tau_{f_k}}\leq\frac{U_k^T+L_k^T+\varepsilon_{f_k}}{2};\ f_k\left(x\right)+\kappa\xi_{f_k}\leq L_k^T+\xi_{f_k};\quad for\ k=1,2,\ldots.,p$$

$$where\ \theta=-\log\left(1-\alpha\right),\kappa=\log\gamma,\eta=-\tanh^{-1}\left(2\beta-1\right),\psi=4,\tau_{f_k}=\frac{6}{U_k^F-L_k^F}$$

$$\theta+\kappa+\eta\leq 3;\ \theta\geq\kappa;\theta\geq\eta;\ \theta,\kappa,\eta\in\left[0,1\right];\ g_j\left(x\right)\leq b_j;\ \ x\geq 0,$$

This crisp nonlinear programming problem can be solved by appropriate mathematical algorithm.

5. Solution of Multi-objective Structural Optimization Problem (MOSOP) by Neutrosophic Optimization Technique

To solve the MOSOP (1), step 1 of 4.1 is used .After that according to step to pay off matrix is formulated.

$$\begin{array}{c} \\ X^1 \\ X^2 \end{array}\begin{array}{cc} C\left(X\right) & \delta\left(X\right) \\ \left[\begin{array}{cc} C^*\left(X^1\right) & \delta\left(X^1\right) \\ C\left(X^2\right) & \delta^*\left(X^2\right) \end{array}\right] \end{array}$$

According to step-2 the bound of weight objective $U_{C(X)}^T, L_{C(X)}^T;\ U_{C(X)}^I, L_{C(X)}^I$ and $U_{C(X)}^F, L_{C(X)}^F$ for truth, indeterminacy and falsity membership function respectively. Then $L^T_{C(X)} \le C(X) \le U^T_{C(X)}; L^I_{C(X)} \le C(X) \le U^I_{C(X)}; L^F_{C(X)} \le C(X) \le U^F_{C(X)}$. Similarly the bound of deflection objective are $U^T_{\delta(X)}, L^T_{\delta(X)}; U^I_{\delta(X)}, L^I_{\delta(X)}$ *and* $U^F_{\delta(X)}, L^F_{\delta(X)}$ are respectively for truth, indeterminacy and falsity membership function. Then $L^T_{\delta(X)} \le \delta(X) \le U^T_{\delta(X)}; L^I_{\delta(X)} \le \delta(X) \le U^I_{\delta(X)}; L^F_{\delta(X)} \le \delta(X) \le U^F_{\delta(X)}$. Where

$U^F_{C(X)} = U^T_{C(X)}, L^F_{C(X)} = L^T_{C(X)} + \varepsilon_{C(X)}; L^I_{C(X)} = L^T_{C(X)}, U^I_{C(X)} = L^T_{C(X)} + \varepsilon_{C(X)}$

and $U^F_{\delta(X)} = U^T_{\delta(X)}, L^F_{\delta(X)} = L^T_{\delta(X)} + \xi_{\delta(X)}; L^I_{\delta(X)} = L^T_{\delta(X)}, U^I_{\delta(X)} = L^T_{\delta(X)} + \xi_{\delta(X)}$ such that

$0 < \varepsilon_{C(X)} < \left(U^T_{C(X)} - L^T_{C(X)}\right)$ and $0 < \xi_{\delta(X)} < \left(U^T_{\delta(X)} - L^T_{\delta(X)}\right)$.

Therefore the truth, indeterminacy and falsity membership functions for objectives are

$$T_{C(X)}\left(C(X)\right) = \begin{cases} 1 & if \quad C(X) \le L^T_{C(X)} \\ 1 - \exp\left\{-\psi\left(\dfrac{U^T_{C(X)} - C(X)}{U^T_{C(X)} - L^T_{C(X)}}\right)\right\} & if \quad L^T_{C(X)} \le C(X) \le U^T_{C(X)} \\ 0 & if \quad C(X) \ge U^T_{C(X)} \end{cases}$$

$$I_{C(X)}\left(C(X)\right) = \begin{cases} 1 & if \quad C(X) \le L^T_{C(X)} \\ \exp\left\{\dfrac{\left(L^T_{C(X)} + \xi_{C(X)}\right) - C(X)}{\xi_{C(X)}}\right\} & if \quad L^T_{C(X)} \le C(X) \le L^T_{C(X)} + \xi_{C(X)} \\ 0 & if \quad C(X) \ge L^T_{C(X)} + \xi_{C(X)} \end{cases}$$

$$F_{C(X)}\left(C(X)\right) = \begin{cases} 0 & if \quad C(X) \le L^T_{C(X)} + \varepsilon_{C(X)} \\ \dfrac{1}{2} + \dfrac{1}{2}\tanh\left\{\left(C(X) - \dfrac{\left(U^T_{C(X)} + L^T_{C(X)}\right) + \varepsilon_{C(X)}}{2}\right)\tau_{C(X)}\right\} & if \quad L^T_{C(X)} + \varepsilon_{C(X)} \le C(X) \le U^T_{C(X)} \\ 1 & if \quad C(X) \ge U^T_{C(X)} \end{cases}$$

$$\textit{where } 0<\varepsilon_{C(X)},\xi_{C(X)}<\left(U^T_{C(X)}-L^T_{C(X)}\right)$$

and

$$T_{\delta(X)}\left(\delta\left(X\right)\right)=\begin{cases}1 & \textit{if } \delta\left(X\right)\le L^T_{\delta(X)}\\ 1-\exp\left\{-\psi\left(\dfrac{U^T_{\delta(X)}-\delta\left(X\right)}{U^T_{\delta(X)}-L^T_{\delta(X)}}\right)\right\} & \textit{if } L^T_{\delta(X)}\le\delta\left(X\right)\le U^T_{\delta(X)}\\ 0 & \textit{if } \delta\left(X\right)\ge U^T_{\delta(X)}\end{cases}$$

$$I_{\delta(X)}\left(\delta\left(X\right)\right)=\begin{cases}1 & \textit{if } \delta\left(X\right)\le L^T_{\delta}\\ \exp\left\{\dfrac{\left(L^T_{\delta(X)}+\xi_{\delta(X)}\right)-\delta\left(X\right)}{\xi_{\delta(X)}}\right\} & \textit{if } L^T_{\delta(X)}\le\delta\left(X\right)\le L^T_{\delta(X)}+\xi_{\delta(X)}\\ 0 & \textit{if } \delta\left(X\right)\ge L^T_{\delta(X)}+\xi_{\delta(X)}\end{cases}$$

$$F_{\delta(X)}\left(\delta\left(X\right)\right)=\begin{cases}0 & \textit{if } \delta\left(X\right)\le L^T_{\delta(X)}+\varepsilon_{\delta(X)}\\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(\delta\left(X\right)-\dfrac{\left(U^T_{\delta(X)}+L^T_{\delta(X)}\right)+\varepsilon_{\delta(X)}}{2}\right)\tau_{\delta(X)}\right\} & \textit{if } L^T_{\delta(X)}+\varepsilon_{\delta(X)}\le\delta\left(X\right)\le U^T_{\delta(X)}\\ 1 & \textit{if } \delta\left(X\right)\ge U^T_{\delta(X)}\end{cases}$$

where ψ,τ are non-zero parameters prescribed by the decision maker and for

$$\textit{where } 0<\varepsilon_{\delta(X)},\xi_{\delta(X)}<\left(U^T_{\delta(X)}-L^T_{\delta(X)}\right)$$

According to neutrosophic optimization technique considering truth, indeterminacy and falsity membership function for MOSOP (1), crisp non-linear programming problem can be formulated as

$$\textit{Maximize } \left(\alpha+\gamma-\beta\right) \tag{4}$$

Subject to

$$T_{C(X)}\left(C\left(X\right)\right)\ge\alpha;\ T_{\delta(X)}\left(\delta\left(X\right)\right)\ge\alpha;$$

$$I_{C(X)}\left(C\left(X\right)\right)\ge\gamma;\ I_{\delta(X)}\left(\delta\left(X\right)\right)\ge\gamma;$$

$$F_{C(X)}\left(C\left(X\right)\right)\le\beta;\ F_{\delta(X)}\left(\delta\left(X\right)\right)\le\beta;$$

$\sigma_i(X) \le [\sigma_i(X)];$

$\alpha + \beta + \gamma \le 3;\ \alpha \ge \beta;\ \alpha \ge \gamma;$

$\alpha, \beta, \gamma \in [0,1],\ X^{\min} \le X \le X^{\max}$

which is reduced to equivalent non linear programming problem as

$$Maximize\ (\theta + \kappa - \eta) \tag{5}$$

Such that

$$C(X) + \theta \frac{\left(U^T_{C(X)} - L^T_{C(X)}\right)}{\psi} \le U^T_{C(X)};$$

$$C(X) + \frac{\eta}{\tau_{C(X)}} \le \frac{U^T_{C(X)} + L^T_{C(X)} + \varepsilon_{C(X)}}{2};$$

$$C(X) + \kappa\xi_{C(X)} \le L^T_{C(X)} + \xi_{C(X)};$$

$$\delta(X) + \theta \frac{\left(U^T_{\delta(X)} - L^T_{\delta(X)}\right)}{\psi} \le U^T_{\delta(X)};$$

$$\delta(X) + \kappa\xi_{\delta(X)} \le L^T_{\delta(X)} + \xi_{\delta(X)};$$

$$\delta(X) + \frac{\eta}{\tau_{\delta(X)}} \le \frac{U^T_{\delta(X)} + L^T_{\delta(X)} + \varepsilon_{\delta(X)}}{2};$$

$$\sigma_i(X) \le [\sigma_i(X)];$$

$$\theta + \kappa - \eta \le 3;$$

$$\theta \ge \kappa; \theta \ge \eta;$$

$$\theta, \kappa, \eta \in [0,1]\ X^{\min} \le X \le X^{\max}$$

where $\theta = -\ln(1-\alpha);\ \psi = 4;\ \tau_{C(X)} = \frac{6}{\left(U^F_{C(X)} - L^F_{C(X)}\right)};\ \tau_{\delta(X)} = \frac{6}{\left(U^F_{\delta(X)} - L^F_{\delta(X)}\right)};\ \kappa = \ln\gamma;$

$\eta = -\tanh^{-1}(2\beta - 1).$

Solving the above crisp model (5) by an appropriate mathematical programming algorithm we get optimal solution and hence objective functions i.e structural weight and deflection of the loaded joint will attain Pareto optimal solution.

6. Numerical Illustration

A welded beam (Ragsdell and Philips 1976, Fig. 1) has to be designed at minimum cost whose constraints are shear stress in weld (τ), bending stress in the beam (σ), buckling load on the bar (P), and deflection of the beam (δ). The design variables are $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} h \\ l \\ t \\ b \end{bmatrix}$ where h is the the weld size, l is the length of the weld t, is the depth of the welded beam, b is the width of the welded beam.

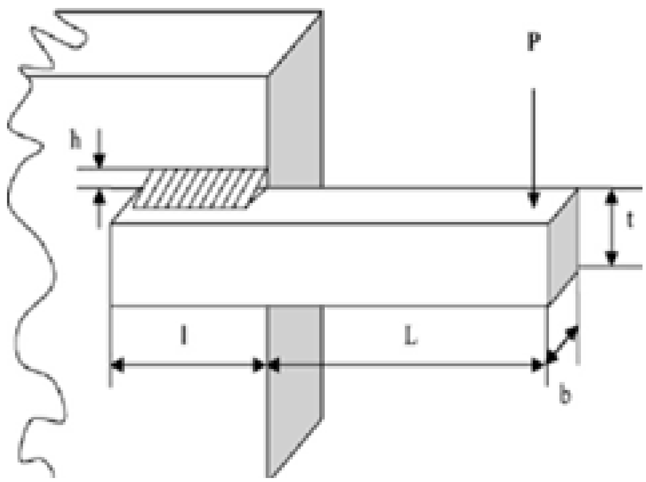

Fig. 1. Design of the welded beam

Cost Function

The performance index appropriate to this design is the cost of weld assembly. The major cost components of such an assembly are (i) set up labour cost, (ii) welding labour cost, (iii) material cost.

$C(X) \equiv C_0 + C_1 + C_2$ where, $f(X) =$ cost function; $C_0 =$ set up cost; $C_1 =$ welding labour cost; $C_2 =$ material cost;

Set up cost C_0: The company has chosen to make this component a weldment, because of the existence of a welding assembly line. Furthermore, assume that fixtures for set up and holding of the bar during welding are readily available. The cost C_0 can therefore be ignored in this particular total cost model.

Welding labour cost C_1: Assume that the welding will be done by machine at a total cost of \$10/hr (including operating and maintenance expense). Furthermore suppose that the machine can lay down a cubic inch of weld in 6 min. The labour cost is then

$C_1 = \left(10 \frac{\$}{hr}\right)\left(\frac{1}{60} \frac{\$}{\min}\right)\left(6 \frac{\min}{in^3}\right) V_w = 1\left(\frac{\$}{in^3}\right) V_w$. Where $V_w =$ weld volume, in^3

Material cost C_2 : $C_2 = C_3V_w + C_4V_B$. Where $C_3 =$ cost per volume per weld material. \$/in^3 $= (0.37)(0.283)$; $C_4 =$ cost per volume of bar stock. \$/in^3 $= (0.37)(0.283)$; $V_B =$ volume of bar, in^3. From geometry $V_w = h^2l$; volume of the weld material (in^3) $V_{weld} = x_1^2x_2$ and $V_B = tb(L+l)$; volume of bar (in^3) $V_{bar} = x_3x_4(L+x_2)$. Therefore cost function become $C(X) = h^2l + C_3h^2l + C_4tb(L+l) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0+x_2)$

Engineering Relationship

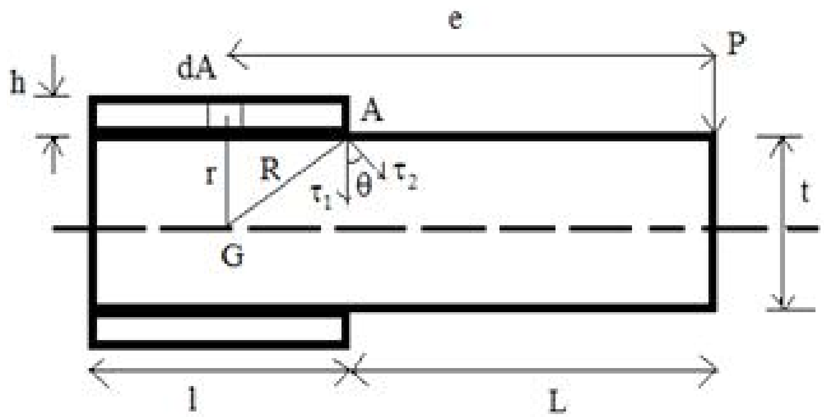

Fig 2. Shear stresses in the weld group.

<u>*Maximum shear stress in weld group:*</u>
To complete the model it is necessary to define important stress states

Direct or primary shear stress $\tau_1 = \dfrac{Load}{Throat\ area} = \dfrac{P}{A} = \dfrac{P}{\sqrt{2}hl} = \dfrac{P}{\sqrt{2}x_1x_2}$

Since the shear stress produced due to turning moment $M = P.e$ at any section is proportional to its radial distance from centre of gravity of the joint 'G', therefore stress due to M is proportional to R and is in a direction at right angles to R. In other words $\dfrac{\tau_2}{R} = \dfrac{\tau}{r} =$ constant. Therefore $R = \sqrt{\left(\dfrac{l}{2}\right)^2 + \left(\dfrac{h+t}{2}\right)^2} = \sqrt{\dfrac{x_2^2}{4} + \dfrac{(x_1+x_3)^2}{4}}$

Where, τ_2 is the shear stress at the maximum distance R and τ is the shear stress at any distance r. Consider a small section of the weld having area dA at a distance r from 'G'. Therefore shear force on this small section $= \tau \times dA$ and turning moment of the shear force about centre of gravity $dM = \tau \times dA \times r = \dfrac{\tau_2}{R} \times dA \times r^2$.

Therefore total turning moment over the whole weld area $M = \dfrac{\tau_2}{R}\int dA \times r^2 = \dfrac{\tau_2}{R}J$. where $J =$ polar moment of inertia of the weld group about centre of gravity. Therefore shear stress due to the turning moment i.e. secondary shear stress, $\tau_2 = \frac{MR}{J}$. In order to find the resultant stress, the primary and secondary shear stresses are combined vectorially. Therefore the maximum resultant shear stress that will be produced at the weld group, $\tau = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos\theta}$, where, $\theta =$ Angle between τ_1 and τ_2. As $\cos\theta = \frac{l/2}{R} = \frac{x_2}{2R}$;

$$\tau = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \frac{x_2}{2R}}.$$

Now the polar moment of inertia of the throat area (A) about the centre of gravity is obtained by parallel axis theorem,

$$J = 2\left[I_{xx} + A + x^2\right] = 2\left[\frac{A\times l^2}{12} + A\times x^2\right] = 2A\left(\frac{l^2}{12} + x^2\right) = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{12} + \frac{(x_1+x_3)^2}{2}\right]\right\}$$

Where, $A =$ throat area $= \sqrt{2}x_1x_2$, $l =$ Length of the weld, $x =$ Perpendicular distance between two parallel axes $= \frac{t}{2} + \frac{h}{2} = \frac{x_1 + x_3}{2}$.

Maximum bending stress in beam:

Now Maximum bending moment $= PL$, Maximum bending stress $= \frac{T}{Z}$, where $T = PL$;

$Z =$ section modulus $= \frac{I}{y}$; $I =$ moment of inertia $= \frac{bt^3}{12}$; $y =$ distance of extreme fibre from centre of gravity of cross section $= \frac{t}{2}$; Therefore $Z = \frac{bt^2}{6}$. So bar bending stress $\sigma(x) = \frac{T}{Z} = \frac{6PL}{bt^2} = \frac{6PL}{x_4x_3^2}$.

Maximum deflection in beam:

Maximum deflection at cantilever tip $= \frac{PL^3}{3EI} = \frac{PL^3}{3E\frac{bt^3}{12}} = \frac{4PL^3}{Ebt^3}$

Buckling load of beam:

buckling load can be approximated by $P_C(x) = \frac{4.013\sqrt{EIC}}{l^2}\left(1 - \frac{a}{l}\sqrt{\frac{El}{C}}\right)$

where, $I =$ moment of inertia $= \frac{bt^3}{12}$; torsional rigidity $C = GJ = \frac{1}{3}tb^3G; l = L; a = \frac{t}{2}$;

$$= \frac{4.013\sqrt{E\frac{t^2b^6}{36}}}{L^2}\left(1-\frac{t}{2L}\sqrt{\frac{E}{4G}}\right) = \frac{4.013\sqrt{EGx_3^6x_4^6/36}}{L^2}\left(1-\frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right);$$

The single-objective optimization problem can be stated as follows

$$Minimize\ \ g(x) \equiv 1.10471x_1^2x_2 + 0.04811(14+x_2)x_3x_4 \tag{6}$$

$$Minimize\ \ \delta(x) \equiv \frac{4PL^3}{Ex_4x_3^2};$$

Such that

$$g_1(x) \equiv \tau(x) - \tau_{\max} \le 0;$$

$$g_2(x) \equiv \sigma(x) - \sigma_{\max} \le 0;$$

$$g_3(x) \equiv x_1 - x_4 \le 0;$$

$$g_4(x) \equiv 0.10471x_1^2x_2 + 0.04811x_3x_4(14+x_2) - 5 \le 0;$$

$$g_5(x) \equiv 0.125 - x_1 \le 0;$$

$$g_6(x) \equiv \delta(x) - \delta_{\max} \le 0;$$

$$g_7(x) \equiv P - P_C(x) \le 0;$$

$$0.1 \le x_1, x_4 \le 2.0$$

$$0.1 \le x_2, x_3 \le 2.0$$

where $\tau(x) = \sqrt{\tau_1^2 + 2\tau_1\tau_2\frac{x_2}{2R} + \tau_2^2}$; $\tau_1 = \frac{P}{\sqrt{2}x_1x_2}$; $\tau_2 = \frac{MR}{J}$; $M = P\left(L + \frac{x_2}{2}\right)$;

$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1+x_3}{2}\right)^2}$; $J = \left\{\frac{x_1x_2}{\sqrt{2}}\left[\frac{x_2^2}{12} + \left(\frac{x_1+x_3}{2}\right)^2\right]\right\}$; $\sigma(x) = \frac{6PL}{x_4x_3^2}$; $\delta(x) = \frac{4PL^3}{Ex_4x_3^2}$;

$P_C(x) = \frac{4.013\sqrt{EGx_3^6x_4^6/36}}{L^2}\left(1-\frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$; $P =$ Force on beam ; $L =$ Beam length beyond weld; $x_1 =$ Height of the welded beam; $x_2 =$ Length of the welded beam; $x_3 =$ Depth of the welded beam; $x_4 =$ Width of the welded beam; $\tau\left(x\right)=$ Design shear stress; $\sigma\left(x\right)=$ Design normal stress for beam material; $M=$ Moment of P about the centre of gravity of the weld , $J=$ Polar moment of inertia of weld group; $G=$ Shearing modulus of Beam Material; $E=$ Young modulus; $\tau_{\max}=$ Design Stress of the weld; $\sigma_{\max}=$ Design normal stress for the beam material; $\delta_{\max}=$ Maximum deflection; $\tau_1=$ Primary stress on weld throat. $\tau_2=$ Secondary torsional stress on weld. Input data are given in table 1.

Table 1. Input data for crisp model (6)

Applied load P (lb)	Beam length beyond weld L (in)	Young Modulus E (psi)	Value of G (psi)	Maximum allowable shear stress $\tau_{\max}$ (psi)	Maximum allowable normal stress $\sigma_{\max}$ (psi)
6000	14	3×10^6	12×10^6	13600 with fuzzy region 50	30000 with fuzzy region 50

Solution: : According to step 2 of 4.1 pay-off matrix is formulated as follows

$$\begin{array}{c} \\ X^1 \\ X^2 \end{array}\begin{array}{cc} C\left(X\right) & \delta\left(X\right) \\ \left[\begin{array}{c} 7.700387 \\ 11.91672 \end{array}\right. & \left.\begin{array}{c} 0.2451363 \\ 0.1372000 \end{array}\right] \end{array}$$

Here

$U^F_{C(X)}=U^T_{C(X)}=11.91672, L^F_{C(X)}=L^T_{C(X)}+\varepsilon_1=7.700387+\varepsilon_1;$

$L^I_{C(X)}=L^T_{C(X)}=7.700387, U^I_{C(X)}=L^T_{C(X)}+\xi_1=7.700387+\xi_1$

such that $0<\varepsilon_1,\xi_1<\left(11.91672-7.700387\right)$;

$U^F_{\delta(X)}=U^T_{\delta(X)}=0.2451363, L^F_{\delta(X)}=L^T_{\delta(X)}+\varepsilon_2=.1372000+\varepsilon_2;$

$L^I_{\delta(X)}=L^T_{\delta(X)}=0.1372000, U^I_{\delta(X)}=L^T_{\delta(X)}+\xi_2=0.1372000+\xi_2$

such that $0<\varepsilon_2,\xi_2<\left(0.2451363-0.1372000\right)$

Here truth, indeterminacy, and falsity membership function for objective functions $C\left(X\right),\delta\left(X\right)$ are defined as follows

$$T_{C(X)}\left(C(X)\right)=\begin{cases}1 & if\ C(X)\le 7.700387\\ 1-\exp\left\{-4\left(\dfrac{11.91672-C(X)}{4.216333}\right)\right\} & if\ \ 7.700387\le C(X)\le 11.91672\\ 0 & if\ \ C(X)\ge 11.91672\end{cases}$$

$$I_{C(X)}\left(C(X)\right)=\begin{cases}1 & if\ C(X)\le 7.700387\\ \exp\left\{\dfrac{(7.700387+\xi_1)-C(X)}{\xi_1}\right\} & if\ \ 7.700387\le C(X)\le 7.700387+\xi_1\\ 0 & if\ \ C(X)\ge 7.700387+\xi_1\end{cases}$$

$$F_{C(X)}\left(C(X)\right)=\begin{cases}0 & if\ C(X)\le 7.700387\\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(C(X)-\dfrac{19.617107+\varepsilon_1}{2}\right)\dfrac{6}{\left(4.216333-\varepsilon_1\right)}\right\} & if\ \ 7.700387\le C(X)\le 11.91672\\ 1 & if\ \ C(X)\ge 11.91672\end{cases}$$

$0<\varepsilon_1,\xi_1<4.216333$

and

$$T_{\delta(X)}\left(\delta(X)\right)=\begin{cases}1 & if\ \delta(X)\le 0.1372000\\ 1-\exp\left\{-4\left(\dfrac{0.2451363-\delta(X)}{0.1079363}\right)\right\} & if\ \ 0.1372000\le \delta(X)\le 0.2451363\\ 0 & if\ \ \delta(X)\ge 0.2451363\end{cases}$$

$$I_{\delta(X)}\left(\delta(X)\right)=\begin{cases}1 & if\ \delta(X)\le 0.1372000\\ \exp\left\{\dfrac{(0.1372000+\xi_2)-\delta(X)}{\xi_2}\right\} & if\ \ 0.1372000\le \delta(X)\le 0.1372000+\xi_2\\ 0 & if\ \ \delta(X)\ge 0.1372000+\xi_2\end{cases}$$

$$F_{\delta(X)}\left(\delta(X)\right)=$$
$$\begin{cases} 0 & \text{if } \delta(X)\le 0.1079363+\varepsilon_2 \\ \frac{1}{2}+\frac{1}{2}\tanh\left\{\left(\delta(X)-\frac{0.3823363+\varepsilon_2}{2}\right)\frac{6}{0.1079363-\varepsilon_2}\right\} & \text{if } 0.1079363+\varepsilon_2\le\delta(X)\le 0.2451363 \\ 1 & \text{if } \delta(X)\ge 0.2451363 \end{cases}$$

$0<\varepsilon_2,\xi_2<0.1079363$

According to neutrosophic optimization technique the MOSOP (6) can be formulated as

$$\text{Maximize } \left(\theta+\kappa-\eta\right) \tag{7}$$

$$1.10471x_1^2x_2+0.04811\left(14+x_2\right)x_3x_4+\frac{4.216333}{4}\theta\le 11.91672;$$

$$1.10471x_1^2x_2+0.04811\left(14+x_2\right)x_3x_4+\frac{\eta\left(4.216333-\varepsilon_1\right)}{6}\le\frac{\left(19.617107+\varepsilon_1\right)}{2};$$

$$1.10471x_1^2x_2+0.04811\left(14+x_2\right)x_3x_4+\kappa\xi_1\le\left(7.700387+\xi_1\right);$$

$$\frac{4PL^3}{Ex_4x_3^2}+\frac{0.1079363}{4}\theta\le 0.2451363;$$

$$\frac{4PL^3}{Ex_4x_3^2}+\frac{\eta\left(0.1079363-\varepsilon_2\right)}{6}\le\frac{\left(0.3823363+\varepsilon_2\right)}{2};$$

$$\frac{4PL^3}{Ex_4x_3^2}+\kappa\xi_2\le\left(0.1372000+\xi_2\right);$$

$$g_1(x)\equiv\tau(x)-\tau_{\max}\le 0;$$
$$g_2(x)\equiv\sigma(x)-\sigma_{\max}\le 0;$$
$$g_3(x)\equiv x_1-x_4\le 0;$$
$$g_4(x)\equiv 0.10471x_1^2x_2+0.04811x_3x_4\left(14+x_2\right)-5\le 0;$$
$$g_5(x)\equiv 0.125-x_1\le 0;$$
$$g_6(x)\equiv\delta(x)-\delta_{\max}\le 0;$$
$$g_7(x)\equiv P-P_C(x)\le 0;$$

$$0.1 \le x_1, x_4 \le 2.0$$
$$0.1 \le x_2, x_3 \le 2.0$$
$$\theta + \kappa + \eta \le 3; \theta \ge \kappa; \theta \ge \eta$$
$$\theta = -\ln(1-\alpha); \psi = 4; \tau_{C(X)} = \frac{6}{\left(U^F_{C(X)} - L^F_{C(X)}\right)}; \tau_{\delta(X)} = \frac{6}{\left(U^F_{\delta(X)} - L^F_{\delta(X)}\right)}; \kappa = \ln\gamma;$$
$$\eta = -\tanh^{-1}(2\beta - 1).$$
$$\tau(x) = \sqrt{\tau_1^2 + 2\tau_1\tau_2 \frac{x_2}{2R} + \tau_2^2} \; ; \; \tau_1 = \frac{P}{\sqrt{2}x_1x_2}; \tau_2 = \frac{MR}{J}; M = P\left(L + \frac{x_2}{2}\right); R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2} \; ;$$
$$J = \left\{\frac{x_1x_2}{\sqrt{2}}\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}; \; \sigma(x) = \frac{6PL}{x_4x_3^2}; \; \delta(x) = \frac{4PL^3}{Ex_4x_3^2};$$
$$P_C(x) = \frac{4.013\sqrt{EGx_3^6x_4^6/36}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right);$$

Now, using above mentioned truth, indeterminacy and falsity membership function NLP (6) can be solved by NSO technique for different values of $\varepsilon_{g(x)}, \varepsilon_{\delta(x)}$ and $\xi_{g(x)}, \xi_{\delta(x)}$. The optimum solution of MOSOP(6) is given in Table 2.

Table 2. Comparison of Optimal solution of MOSOP (6) based on different methods

Methods	x_1 $(inch)$	x_2 $(inch)$	x_3 $(inch)$	x_4 $(inch)$	$C(X)$	$\delta(X)$
Fuzzy single-objective non-linear programming (FSONLP)	1.298580	0.9727729	1.692776	1.298580	3.395620	0.2456363
Intuitionistic Fuzzy single-objective non-linear programming (FSONLP)	1.298580	0.9727730	1.692776	1.298580	3.395620	0.2352203
Neutosophic optimization(NSO) $\varepsilon_{g(x)} = 0.42,\ \xi_{g(x)} = 0.42,$ $\varepsilon_{\delta(x)} = 0.01,\ \xi_{\delta(x)} = 0.01,$	1.957009	1.240976	2	1.957009	8.120387	0.1402140

A detailed comparison has been made among the minimum length, depth, height and width of the weld,welding cost and deflection Also the results have been compared among fuzzy, intuitionistic, neutrosophic optimization technique in perspective of welded beam design in Table 2. It has been observed that Intuitionistic fuzzy nonlinear optimization provides better result in comparison with other mentioned method in this study. However, it may also be noted that the efficiency of the proposed method depends on the model chosen to a greater extent. In the present study it has also been investigated that cost of welding is maximum and deflection is minimum in neutrosophic optimization technique compared to the other method investigated.

7. Conclusions

In this paper, a multi objective neutrosophic optimization algorithm has been developed by defining truth, indeterminacy and falsity membership function which are independent to each other. It has been shown that the developed algorithm can be applied to optimize a multi objective nonlinear structural design. Simulation example, i.e. welded beam design has been provided to illustrate the optimization procedure, effectiveness and advantages of the proposed neutrosophic optimization method. The extension of the proposed optimization can be neutrosophic optimization using ranking method of neutrosophic fuzzy numbers ,considered for height, length, depth and width of weld and applied load as further topics of interest.

Conflict of interests: The authors declare that there is no conflict of interests.

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