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Section A

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On $\overset{0}{g}$-Closed Sets In Topological Spaces

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Abstract

In this paper, we introduce and study a new class of sets namely $\overset{0}{g}$-closed sets which settled in between the class of g*-closed sets[21] and the class of g-closed sets[3] and then we study many basic properties of $\overset{0}{g}$-closed sets together with the relationship of these sets with some other sets. As applications of $\overset{0}{g}$-closes sets, we introduce some new separation properties, namely $T^0_{1/2}$-spaces, ${}^0T_{1/2}$-spaces, $\overset{0}{T}_{1/2}$-spaces and ${}_\alpha\overset{0}{T}_{1/2}$-spaces. Further we introduce and study new types of continuous maps called $\overset{0}{g}$-continuous maps and $\overset{0}{g}$-irresolute maps.

Key words : $\overset{0}{g}$-closed sets; $T^0_{1/2}$-spaces, ${}^0T_{1/2}$-spaces; $\overset{0}{T}_{1/2}$-spaces; ${}_\alpha\overset{0}{T}_{1/2}$-spaces; $\overset{0}{g}$-continuity.

1. Introduction

The study of g-closed sets in a topological space was initiated by Levine[3] in 1970. Levine[1] also introduced the concept of semi-open sets and semi-continuity in a topological spaces in 1963. Bhattacharya and Lahiri[7] introduced sg-closed sets in 1987. Arya and Nour [8] defined gs-closed sets in 1990. N.jasted[2] introduced the concepts of α-closed sets for topological spaces in 1965. Dontchev[16] (resp. Palaniappan and Rao[14], Gnanambal[18]) introduced gsp-closed (resp. rg-closed, gpr-closed) sets in 1995 (resp. 1993, 1997). Veera Kumar introduced $\hat{g}$-closed sets[22], ψ-closed sets[20], *g-closed sets[25], g*-closed sets[21], #g-closed sets[26] and #gs-closed sets[29]. Manojet al. introduced the concepts of $\hat{\hat{g}}$-closed sets[24], **g-closed sets[30] and *gs-closed sets[32] in 2007.

In this paper we study relationships of $\overset{0}{g}$-closed sets with the above mentioned sets. Norman Levine[3], Bhattacharya and Lehiri[7] and Devi *et al.*[12] introduced $T_{1/2}$-spaces, semi-$T_{1/2}$ spaces, T_b-spaces (T_d-spaces, $_\alpha T_b$-spaces, $_\alpha T_d$-spaces) respectively. Veera Kumar introduced T_c-spaces[21], $_\alpha T_c$-spaces[21], T_f-spaces[22], $T^*_{1/2}$-spaces and $^*T_{1/2}$ spaces[21] and semi $T_{1/3}$-spaces[20].

We introduce and study new classes of spaces, namely the class of $T^0_{1/2}$-spaces, the class of $^0T_{1/2}$-spaces, the class of $\overset{0}{T}_{1/2}$-spaces and the class of $_\alpha\overset{0}{T}_{1/2}$ spaces. We also characterize and study some relationships of these spaces with the above defined spaces.We again introduced $\overset{0}{g}$-continuous maps, $\overset{0}{g}$-irresolute maps and investigate some of their properties.

Throughout this paper (X, τ), (Y, σ) and (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless or otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A), pcl(A) and A^c denote the closure of A, the interior of A, pre-closure of A and the complement of A respectively.

2. Preliminaries:

We recall the following definitions which are useful in the sequel.

Definition 2.01 : A subset A of a topological space (X, τ) is called semi-open[1] (resp. semi-closed, pre-open[5], pre-closed, α-open[2], semi-preopen[6], semi-pre closed, regular open, regular closed) if A ⊆ cl(int(A)) (resp. int(cl(A)) ⊆ A, A ⊆ int(cl(A)), cl(int(A)) ⊆ A, A ⊆int(cl(int(A))), A ⊆ cl(int(cl(A))), int(cl(int(A))) ⊆ A, A = int(cl(A)), A = cl(int(A)).

*Definition 2.02 :*A subset A of a topological space (X, τ) is called g-closed[3] (resp. sg-closed[7], gs-closed[8],gp-closed[9], αg-closed[15], rg-closed[14], gpr-closed[18], gsp-closed[16], $\hat{g}$-closed[22], ψ-closed[20], *g-closed[24], $\hat{\hat{g}}$-closed[24], g*-closed[21], *gs-closed[15]) if cl(A) ⊆ U (resp. scl(A) ⊆ U, scl(A) ⊆ U, pcl(A) ⊆ U, αcl(A) ⊆ U, cl(A) ⊆ U, pcl(A) ⊆ U, spcl(A) ⊆ U, cl(A) ⊆ U, scl(A) ⊆ U, cl(A) ⊆ U, cl(A) ⊆ U, cl(A) ⊆ U, scl(A) ⊆ U) whenever A ⊆ U and U is open (resp. semi-open, open, open, open, regular-open, regular open, open, semi-open, sg-open, $\hat{g}$-open, sg-open, g-open, $\hat{g}$-open) in (X, τ).

Definition 2.03 : A function f : (X, τ) → (Y, σ) is called semicontinuous[1] (resp. g-continuous[11], sg-continuous[11], gs-continuous[17], rg-continuous[14], gpr-continuous[18], gsp-continuous[16], ψ-continuous[20], g*-continuous[21], *gs-continuous[15], gp-continuous[13], αg-continuous[18], pre-continuous[5] sp-continuous[6], α-continuous[23], g*-irresolute[21], αg-irresolute[18], gs-irresolute[17]) if f^{-1} (V) is semi-closed (resp. g-closed, sg-closed, gs-closed, rg-closed, gpr-closed, gsp-closed, ψ-closed, g*-closed, *gs-closed, gp-closed, αg-closed, pre-closed, sp-closed, α-closed, g*-closed, αg-closed, gs-closed) set in (X, τ) for every closed (resp. closed, closed, closed, closed, closed, closed, closed, closed, closed, closed, closed, closed, closed, closed, g*-closed, αg-closed, gs-closed) set in (Y, σ).

*Definition 2.04 :*A space (X, τ) is called $T_{1/2}$ [3] (resp. T_b [12], T_c [21], T_d [12], T_f [22], T_s [15], $_\alpha T_b$ [19], $_\alpha T_c$ [21], $_\alpha T_d$ [19], semi-$T_{1/2}$ [7], semi-$T_{1/3}$ [20], $T^*_{1/2}$ [21], $^*T_{1/2}$ [21]) space if every g-closed (resp. gs-closed,gs-closed, gs-closed, g-closed, *gs-closed, αg-closed, αg-closed, αg-closed, sg-closed, ψ-closed, g*-closed, g-closed) set is a closed (resp. closed, g*-closed, g-closed, $\hat{g}$-closed, closed, closed, g*-closed, g-closed, semi-closed, semi-closed, closed, g*-closed) set in (Y, σ).

Definition 2.05 : A subset A of a topological space (X, τ) is called

(i) a δ-closed set[37] if A = cl_δ(A), where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \phi, U \in \tau \text{ and } x \in U\}$.

(ii) a θ-closed set [37] if A = cl_θ(A), where $cl_\theta(A) = \{x \in X : cl(U) \cap A \neq \phi, U \in \tau \text{ and } x \in U\}$.

3. Relationships of $\overset{0}{g}$ –closed sets with other sets

In this section we study the relationship of $\overset{0}{g}$-closed sets with other sets.

*Definition 3.01 :*A subset A of topological space (X, τ) is called $\overset{0}{g}$-closed set if cl(A) ⊆ U whenever A ⊆ U and

U is a g*-open set in (X, τ).

The complement of $\overset{0}{g}$ -closed set is called $\overset{0}{g}$ -open set.

Theorem 3.02 : (i) Every closed, θ-closed, δ-closed and g*-closed set is $\overset{0}{g}$ -closed set.

(ii) Every $\overset{0}{g}$ -closed set is gs-closed, gp-closed, gpr-closed, αg-closed, rg-closed and gsp-closed set.

Next examples show that converse of the above theorem is not true in general.

Example 3.03 : Let X = {a, b, c}, τ = {ϕ,{a, b}, X}. Consider A = {a, c} then A is not a closed set and δ-closed set. However A is a $\overset{0}{g}$ -closed set.

Example 3.04 : Let X = {a, b, c}, τ = {ϕ, {a}, {a, b}, {a, c}, X}. Consider B = {c} then B is not a θ-closed set. However B is a $\overset{0}{g}$ -closed set

Example 3.05 : Let X = {a, b, c}, τ = {ϕ, {b}, {a, c}, X}. Consider C = {a, b} then C is not a g*-closed set. However C is a $\overset{0}{g}$ -closed.

*Example 3.06 :*Let X = {a, b, c,d}, τ = {ϕ, {a}, {a, b, c}, X}. Consider D = {b, c} then D is not a $\overset{0}{g}$ -closed set. However D is gs-closed set and αg-closed set.

*Example 3.07 :*Let X= {a, b, c}, τ = {ϕ, {a}, {c}, {a, c}, X}. Consider E ={a, c} then E is a not a $\overset{0}{g}$ -closed set. However E is rg-closed set and gpr-closed set.

Example 3.08 : Let X = {a, b, c}, τ = {ϕ, {a}, {a, b}, X}. Consider F = {b} then B is a not a $\overset{0}{g}$ -closed set. However B is gp-closed set and gsp-closed set.

Theorem 3.09 : Every $\overset{0}{g}$ -closed set is g-closed set.

Therefore the class of $\overset{0}{g}$ -closed sets is properly contained in the class of g-closed sets and properly contains the class of closed sets, the class of θ-closed sets,the class of δ-closed sets, the class of αg-closed sets, the class of rg-closed sets, the class of gs-closed sets, the class of gp-closed sets, the class of gpr-closed sets, the class of gsp-closed sets, and the class of g*-closed sets.

Remark 3.10 : $\overset{0}{g}$ -closed set is independent from semi-closed sets, sg-closed sets,α-closed sets, ψ-closed sets, pre-closed sets, sp-closed setsand *gs-closed sets.

The following examples support the above results.

Example 3.11 : Let X = {a, b, c, d}, τ = {ϕ, {a}, {b}, {a, b}, {a, d}, {a, b, c}, {a, b, d}, X}. consider A = {b} then A isψ-closed and sg-closed set. However A is not a $\overset{0}{g}$ -closed set.

Example 3.12 : Let X = {a, b, c, d}, τ = {ϕ, {a}, {b}, {a, b, c}, X}. Consider A = {c} then A is semi-closed but not a $\overset{0}{g}$ -closed set.

*Example 3.13 :*Let X = {a, b, c, d}, τ = {ϕ, {a, b}, X}. Consider A = {a, c} then A is $\overset{0}{g}$ -closed set but not, semi-closed, ψ-closed and sg-closed.

Example 3.14 : In example (3.08), A is α-closed and *gs-closed. However A is not $\overset{0}{g}$ -closed.

Example 3.15 : In example (3.03), Consider A={a} then A is p-closed and sp-closed. However A is not $\overset{0}{g}$ -closed.

Example 3.16 : Let X = {a, b, c}, τ = {ϕ, {a}, {b, c}, X}. Consider A = {b} then A is $\overset{0}{g}$ -closed but not α-closed and *gs-closed.

Example 3.17 : Let X = {a, b, c}, τ = {ϕ, {a}, X}. Consider A = {a, c} then A is $\overset{0}{g}$ -closed but not p-closed and sp-closed.

Remark 3.18 : $\overset{0}{g}$ -closed set is not necessarily *g-closed setas in example (3.05),Consider A = {b} then A is $\overset{0}{g}$ -closed but not *g-closed.

The following diagram shows the relationships of $\overset{0}{g}$ -closed set with other sets.

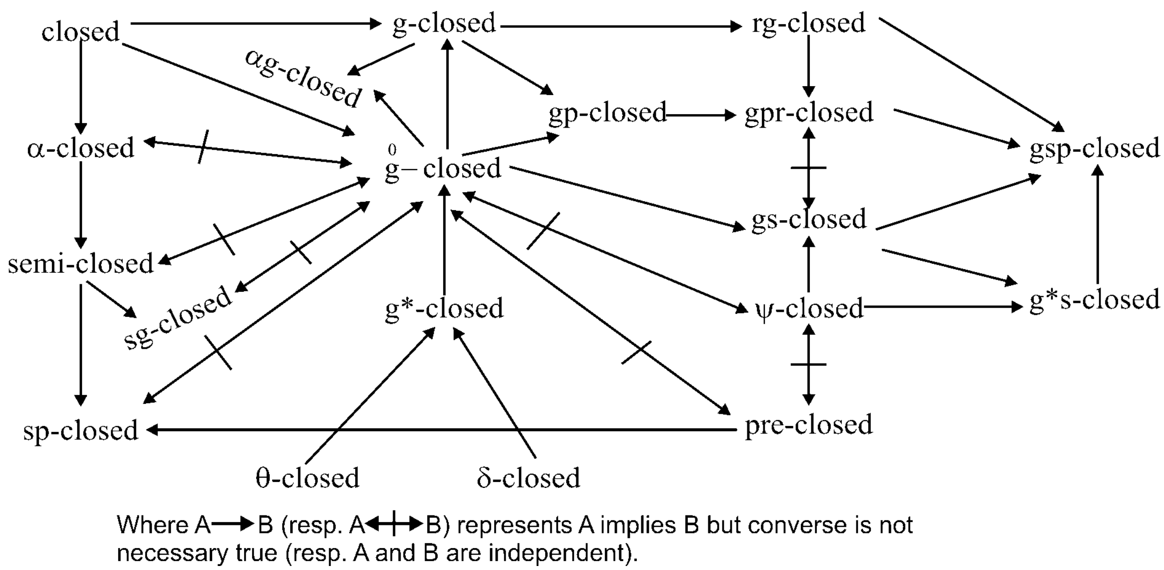

Diagram (3.19)

4. Basic properties of $\overset{0}{g}$ -closed sets :

In this section we introduce the following theorems and definitions.

*Theorem 4.01 :*Union of two $\overset{0}{g}$ -closed sets is again $\overset{0}{g}$-closed set.

*Remark 4.02 :*Intersection of two $\overset{0}{g}$-closed set is not necessarily $\overset{0}{g}$-closed set as in example (3.17), Consider A = {a, b} and B ={a, c} then A∩B = {a} which is not a $\overset{0}{g}$-closed set.

*Theorem 4.03 :*If A is a $\overset{0}{g}$-closed set in a space (X, τ) and A ⊆ B ⊆ cl(A) then B is also a $\overset{0}{g}$-closed set.

Theorem 4.04 : A is $\overset{0}{g}$-closed set of (X, τ) if and only if cl(A) - A does not contain any non-empty g*-closed set.

*Theorem 4.05 :*Every $\overset{0}{g}$-closed set is closed if and only if every singleton of X is either open or g*-closed.

*Definition 4.06 :*Let (X, τ) be a topological space and A⊆ X. We define the $\overset{0}{g}$-closure of A (briefly $\overset{0}{g}$-cl(A)) to be the intersection of all $\overset{0}{g}$-closed sets containing A. In symbols, $\overset{0}{g}$-cl(A) = ∩ {B : A ⊆ B and B ∈ $\overset{0}{g}$-closed sets}.

*Theorem 4.07 :*Let (X, τ) be a topological space and A ⊆ X. The following properties are hold.

(i) $\overset{0}{g}$-cl(A) is the smallest $\overset{0}{g}$-closed set containing A and

(ii) A is $\overset{0}{g}$-closed iff $\overset{0}{g}$-cl(A) = A.

*Theorem 4.08 :*For any two subsets A and B of (X, τ)

(i) $A \subseteq B$ then $\overset{0}{g}$-cl(A) $\subseteq$ $\overset{0}{g}$-cl(B).

(ii) $\overset{0}{g}$-cl($A \cap B$) $\subseteq$ $\overset{0}{g}$-cl(A) $\cap$ $\overset{0}{g}$-cl(B).

*Theorem 4.09 :*If A is a $\overset{0}{g}$-closed set and B is a closed set, then $A \cap B$ is a $\overset{0}{g}$-closed set.

*Theorem 4.10 :*If $B \subseteq A \subseteq X$, B is a $\overset{0}{g}$-closed set relative to A and A is open and $\overset{0}{g}$-closed in (X, τ). Then B is $\overset{0}{g}$-closed in (X, τ).

*Definition 4.11 :*Let (X, τ) be a topological space and $A \subseteq X$. We define the $\overset{0}{g}$-interior of A (briefly $\overset{0}{g}$-int(A)) to be the union of all $\overset{0}{g}$-open sets contained in A.

*Theorem 4.12 :*For any $A \subseteq X$, int(A) $\subseteq$ $\overset{0}{g}$-int(A) $\subseteq$ A.

5. Applications of $\overset{0}{g}$-closed sets

In this section we introduce the following definitions.

*Definition 5.01 :*A topological space (X, τ) is called $T^0_{1/2}$ -space if every $\overset{0}{g}$-closed set in it is clsoed.

*Theorem 5.02 :*Every $T_{1/2}$-space is $T^0_{1/2}$ -space.

*Theorem 5.03 :*Every T_b-space is $T^0_{1/2}$ -space.

The space in example (3.07) supports that the converse of the above theorem is not true.

*Theorem 5.04 :*Every $T^0_{1/2}$ -space is $T^*_{1/2}$-space.

The space in example (3.17) supports that the converse of the above theorem is not true.

*Theorem 5.05 :*Every ${}_{\alpha}T_b$-space is $T^0_{1/2}$ -space.

Therefore the class of $T^0_{1/2}$ -spacesproperly contains the class of $T_{1/2}$-spaces, T_b-spaces and ${}_{\alpha}T_b$-spaces and is properly contained in the class of $T^*_{1/2}$-spaces.

Remark 5.06 : $T^0_{1/2}$ -space and T_s-space are independent from each other.The space in example (3.07) is $T^0_{1/2}$ -space but not T_s-space.The space in example (3.05) is T_s-space but not-space.

Remark 5.07 : semi-$T_{1/2}$ spaces and semi-$T_{1/3}$ spaces are not necessarily $T^0_{1/2}$ -spaces.

The space in example (3.17) is a semi-$T_{1/2}$ space and semi-$T_{1/3}$ space but not a $T^0_{1/2}$ -space.

*Theorem 5.08 :*A space (X, τ) is $T^0_{1/2}$ -space if and only if every singleton of X is either open or g*-closed.

*Definition 5.09 :*A topological space (X, τ) is called $T^0_{1/2}$ -space if every g-closed set in it is -clsoed set.

*Theorem 5.10 :*Every $T_{1/2}$-space is $T^0_{1/2}$ -space.

The space in example (3.12) supports that the converse of the above theorem is not true.

Theorem 5.11 : Every $^*T_{1/2}$-space is $T^0_{1/2}$ -space.

The space in example (3.17) supports that the converse of the above theorem is not true.

Remark 5.12 : $T^0_{1/2}$ -space is not necessarily T_s-space as it can be seen from the following example.

Example 5.13 : Let X = {a, b, c}, $\tau = \{\phi, X\}$. Then (X, τ) is $T^0_{1/2}$ -space. However it is not T_f-space.

*Theorem 5.14 :*A space (X, τ) is $T_{1/2}$-space if and only if it is $T^0_{1/2}$ -space and ${}^0T_{1/2}$ -space.

*Theorem 5.15 :*If (X, τ) is a ${}^0T_{1/2}$ -space then for each $x \in X$, {x} is either closed or $\overset{0}{g}$ -closed.

Remark 5.16 : ${}^0T_{1/2}$ -space is not necessarily $T^0_{1/2}$ -space.

The space in example (3.05) is a ${}^0T_{1/2}$ -space but not a $T^0_{1/2}$ -space.

*Definition 5.17 :*A topological space (X, τ) is called $\overset{0}{T}_{1/2}$ -space if every gs-closed set in it is $\overset{0}{g}$-clsoed set.

Theorem 5.18 : Every T_b-space and T_c-space is $\overset{0}{T}_{1/2}$ -space.

The space in example (3.03) is $\overset{0}{T}_{1/2}$ -space but not T_b-space and the space in example (3.05) is $\overset{0}{T}_{1/2}$ -space but not T_c-space.

*Theorem 5.19 :*Every $\overset{0}{T}_{1/2}$ -space is T_d-space.

*Theorem 5.20 :*Every $\overset{0}{T}_{1/2}$ -space is ${}^0T_{1/2}$ -space.

The space in example (3.07) supports that the converse of the above theorem is not true.

Therefore the class of $\overset{0}{T}_{1/2}$ -spaces properly contains the class of T_b-spaces and the class of T_c-spaces and is properly contained in the class of T_d-spaces and ${}^0T_{1/2}$-spaces.

*Theorem 5.21 :*If (X, τ) is $T_{1/2}$-space and T_d-space then it is $\overset{0}{T}_{1/2}$ -space.

*Theorem 5.22 :*A space (X, τ) is $T^0_{1/2}$-space and $\overset{0}{T}_{1/2}$-space if and only if it is T_b-space.

Remark 5.23 : $\overset{0}{T}_{1/2}$-space and $T_{1/2}$-space are independent from each other. In example (3.05) the space (X, τ) is $\overset{0}{T}_{1/2}$-space but not $T_{1/2}$-space and in example (3.07) the space (X, τ) is $T_{1/2}$-space but not $\overset{0}{T}_{1/2}$-space.

Definition 5.24 : A topological space (X, τ) is called ${}_\alpha\overset{0}{T}_{1/2}$-space if every αg-closed set in it is $\overset{0}{g}$-clsoed set.

*Theorem 5.25 :*Every ${}_\alpha T_b$-space and ${}_\alpha T_c$-space is ${}_\alpha\overset{0}{T}_{1/2}$-space.

The space in example (3.17) is ${}_\alpha\overset{0}{T}_{1/2}$-space but not ${}_\alpha T_b$-space and ${}_\alpha T_c$-space.

Theorem 5.26 : Every ${}_\alpha\overset{0}{T}_{1/2}$-space is ${}_\alpha T_d$-space.

Theorem 5.27 : Every ${}_{\alpha}\overset{0}{T}_{1/2}$-space is ${}^{0}T_{1/2}$-space.

The space in example (3.08) is ${}^{0}T_{1/2}$-space but not ${}_{\alpha}\overset{0}{T}_{1/2}$-space and ${}_{\alpha}T_{c}$-space.

Therefore the class of ${}_{\alpha}\overset{0}{T}_{1/2}$-spaces properly contains the class of ${}_{\alpha}T_{b}$-spaces and the class of ${}_{\alpha}T_{c}$-spaces and properly contained in the class of ${}_{\alpha}T_{d}$-spaces and the class of ${}^{0}T_{1/2}$-spaces.

Theorem 5.28 : If (X, τ) is an ${}_{\alpha}\overset{0}{T}_{1/2}$-space then each singleton of X is either closed or $\overset{0}{g}$-open.

Theorem 5.29 : A space (X, τ) is an ${}_{\alpha}T_{b}$-space if and only if it $T^{0}_{1/2}$-space and ${}_{\alpha}\overset{0}{T}_{1/2}$-space.

*Theorem 5.30 :*If (X, τ) is ${}^{0}T_{1/2}$-space and ${}_{\alpha}T_{d}$-space then it is ${}_{\alpha}\overset{0}{T}_{1/2}$-space.

The following diagram shows the relationships between the separation axioms discussed in this section.

$\overset{0}{T} + T^{0}_{1/2} \Leftrightarrow T_{b}$, $\alpha\overset{0}{T}_{1/2} + \overset{0}{T}_{1/2} \Leftrightarrow \alpha T_{b}$, $\alpha T_{d} + {}^{0}T_{1/2} \Rightarrow \alpha\overset{0}{T}_{1/2}$

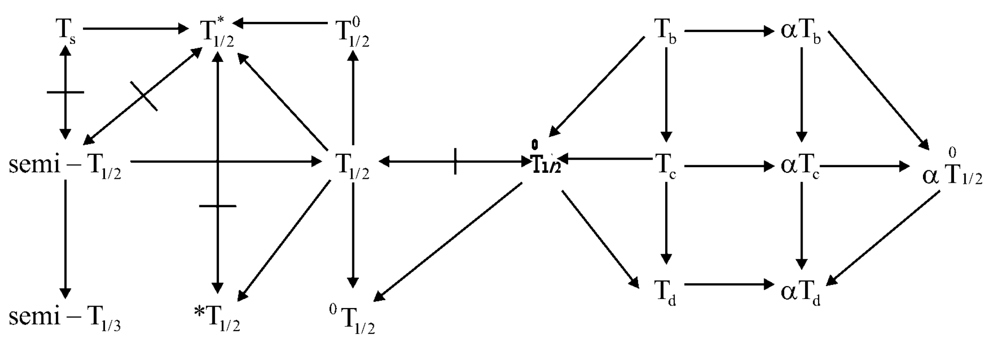

Diagram (5.31)

6. $\overset{0}{g}$ *-Continuous and* $\overset{0}{g}$ *-irresolute maps*

In this section we introduce the following definitions.

*Definition 6.01 :*A map f : (X, τ) → (Y, σ) is said to be $\overset{0}{g}$ -continuous if the inverse image of every σ-closed set in Y is $\overset{0}{g}$ -closed in X.

Theorem 6.02 : Every continuous map and g*-continuous map is $\overset{0}{g}$ -continuous map.

Next examples show that converse of the above theorem is not true in general.

Example 6.03 : Let X = Y = {a, b, c}, τ = {ϕ, {a}, {a, b}, X} and σ = {ϕ, {a}, {a, c}, Y}. Define f : (X, τ) → (Y, σ) by f(a) = c, f(b) = a and f(c) = b then f is not continuous. However f is $\overset{0}{g}$ -continuous.

*Example 6.04 :*Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, {b}, {c}, {b, c}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is not g*-continuous. However f is $\overset{0}{g}$-continuous.

Theorem 6.05 : Every $\overset{0}{g}$-continuous map is (i)αg-continuous, (ii)gs-continuous,(iii) gp-continuous, (iv) rg-continuous, (v) gpr-continuous, (vi) gsp-continuous.

Next examples show that converse of the above theorem is not true in general.

*Example 6.06 :*Let X = Y = {a, b, c, d}, τ = {ϕ, {a}, {a, b}, {a, b, c}, X} and σ = {ϕ, {a, d}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is not $\overset{0}{g}$-continuous. However f is αg-continuous and gs-continuous.

Example 6.07 : Let X = Y = {a, b, c}, τ = {ϕ, {a}, {c}, {a, c}, X} and σ = {ϕ, {b}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is not $\overset{0}{g}$-continuous. However f is rg-continuous and gpr-continuous.

*Example 6.08 :*Let X = Y = {a, b, c}, τ = {ϕ, {a}, {a, b}, X} and σ = {ϕ, {a, c}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is not $\overset{0}{g}$-continuous. However f is gpr-continuous and gsp-continuous.

*Theorem 6.09 :*Every $\overset{0}{g}$-continuous map is g-continuous map.

Remark 6.10 : $\overset{0}{g}$-continuity is independent from (i) semi-continuuity, (ii) sg-continuity, (iii) α-continuity,(iv) pre-continuity,(v) sp-continuity,(vi) Ψ-continuity, (vii) *gs-continuity.

Next examples support the above result.

*Example 6.11 :*Let X = Y = {a, b, c, d}, τ = {ϕ, {a, b}, X} and σ = {ϕ, {d}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is $\overset{0}{g}$-continuousbut notsemi-continuous, sg-continuous and Ψ-continuous.

*Example 6.12 :*Let X = Y = {a, b, c, d}, τ = {ϕ, {a}, {b}, {a, b}, {a, d}, {a, b, c}, {a, b, d}, X} and σ = {ϕ, {b, c}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is sg-continuous and Ψ-continuous but not $\overset{0}{g}$-continuous.

*Example 6.13 :*Let X = Y = {a, b, c}, τ = {ϕ, {a}, {a, b}, X} and σ = {ϕ, {a, c}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is semi-continuous, α-continuous,pre-continuous,sp-continuous and *gs-continuous but not $\overset{0}{g}$-continuous.

*Example 6.14 :*Let X = Y = {a, b, c}, τ = {ϕ, {a}, X} and σ = {ϕ, {c}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is $\overset{0}{g}$-continuous but notpre-continuous,sp-continuous,α-continuous.

*Example 6.15 :*Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, {b}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is $\overset{0}{g}$-continuous but not *gs-continuous.

Therefore the class of $\overset{0}{g}$-continuous maps properly contains the class of continuous maps and the class of g*-continuous maps and it is properly contained in the class of g-continuous maps, the class of αg-continuous maps, the class of gs-continuous maps, the class of gp-continuous maps, the class ofrg-continuous maps,the class of gpr-continuous maps and the class of gsp-continuous maps.

*Definition 6.16 :*A map f : (X, τ) → (Y, σ) is said to be $\overset{0}{g}$-irresolute if the inverse image of every $\overset{0}{g}$-closed set in Y is $\overset{0}{g}$-closed set in X.

*Theorem 6.17 :*Every $\overset{0}{g}$-irresolute map is $\overset{0}{g}$-continuous map.

The converse of the above theorem is not true as it can be seen from the following example.

*Example 6.18 :*Let X = Y = {a, b, c}, τ = {ϕ, {a}, X} and σ = {ϕ, {a}, {c}, {a, c}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is $\overset{0}{g}$-continuous but not $\overset{0}{g}$-irresolute.

*Theorem 6.19 :*Let (X, τ), (Y, σ) and (Z, η) be any three topological spaces. Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) be any two functions then gof : (X, τ) → (Z, η) is

(i) $\overset{0}{g}$-continuous if g is continuous and f is $\overset{0}{g}$-continuous.

(ii) $\overset{0}{g}$-irresolute if g is $\overset{0}{g}$-irresolute and f is $\overset{0}{g}$-irresolute.

(iii) $\overset{0}{g}$-continuous if g is -continuous and f is $\overset{0}{g}$-irresolute.

*Theorem 6.20 :*Let f : (X, τ) → (Y, σ) be a $\overset{0}{g}$-continuous (resp. αg-continuous, g-continuous, gs-continuous) map. If (X, τ) is $T^{0}_{1/2}$ (resp. D, ${}^{0}T_{1/2}$, ${}_{\alpha}\overset{0}{T}_{1/2}$) space then f is continuous (resp. $\overset{0}{g}$-continuous, $\overset{0}{g}$-continuous, $\overset{0}{g}$-continuous) map.

Theorem 6.21 : Let f : (X, τ) → (Y, σ) be a g*-irresolute and closed map then for every $\overset{0}{g}$-closed set A of (X, τ), f(A) is $\overset{0}{g}$-closed set of (Y, σ).

*Theorem 6.22 :*Let f : (X, τ) → (Y, σ) be a onto, $\overset{0}{g}$-irresolute and closed. If (X, τ) is $T^{0}_{1/2}$ -space then (Y, σ) is also $T^{0}_{1/2}$ -space.

Definition 6.23 : A map f : (X, τ) → (Y, σ) is said to be pre-$\overset{0}{g}$-closed if f(A) is a $\overset{0}{g}$-closed set of (Y, σ) for every $\overset{0}{g}$-closed set of (X, τ).

Theorem 6.24 : Let f : (X, τ) → (Y, σ) be a onto, $\overset{0}{g}$-irresolute and pre-$\overset{0}{g}$-closed. If (X, τ) is ${}^{0}T_{1/2}$-space then (Y, σ) is also ${}^{0}T_{1/2}$-space.

*Theorem 6.25 :*Let f : (X, τ) → (Y, σ) be a onto, αg-irresolute and pre-$\overset{0}{g}$-closed. If (X, τ) is D-space then (Y, σ) is also D-space.

Theorem 6.26 : Let f : (X, τ) → (Y, σ) be a onto, gs-irresolute and pre-$\overset{0}{g}$-closed. If (X, τ) is $\overset{0}{T}_{1/2}$-space then (Y, σ) is also $\overset{0}{T}_{1/2}$-space.

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