ISSN 2231-346X (Print)
JUSPS-A Vol. 29(6), 220-242 (2017). Periodicity-Monthly
Section A
ISSN 2319-8044 (Online)

Estd. 1989
JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Mathematics
website:- www.ultrascientist.org

Optimization of Welded Beam with Imprecise Load and Stress by Parameterized Neutrosophic Optimization Technique

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http://dx.doi.org/10.22147/jusps-A/290702

Acceptance Date 30th May, 2017, Online Publication Date 2nd July, 2017

Abstract

This paper develops a solution procedure of Neutrosophic Optimization (NSO) to solve optimum welded beam design with inexact co-efficient and resources. Interval approximation method is used here to convert the imprecise co-efficient which is a triangular neutrosophic number to an interval number. We transform this interval number to a parametric interval valued functional form and then solve this parametric problem by NSO technique. Usually interval valued optimization consist of two level mathematical programs, but a parametric interval valued optimization in neutrosophic environment is direct approach to find the objective function, this is the main advantage. In this paper we have considered a welded beam design with cost of welding as objective and the maximum shear stress in the weld group, maximum bending stress in the beam, buckling load of the beam and deflection at the tip of a welded steel beam as constraints .Numerical example is given here to illustrate this structural model through this approximation method.

Keywords : Single valued Neutrosophic set; Generalized Neutrosophic Number; Nearest Interval Approximation; Interval Valued Function; Neutrosophic Optimization; Welded Beam Optimization.

Subject classification code:90C30,90C70,90C90

1. Introduction

Welding, a process of joining metallic parts with the application of heat or pressure or the both, with or without added material, is an economical and efficient method for obtaining permanent joints in the metallic parts. This welded joints are generally used as a substitute for riveted joint or can be used as an alternative method for casting or forging. The welding processes can broadly be classified into following two groups, the welding process that uses heat alone to join two metallic parts and the welding process that uses a combination of heat and pressure for joining (Bhandari. V. B). However, above all the design of welded beam should preferably be economical and durable one. Since decades, deterministic optimization has been widely used in practice for optimizing welded connection design. These include mathematical traditional optimization algorithms (Ragsdell & Phillips[1]) such as GA-based methods (Deb[2], Deb[3], Coello[4], Coello[5]), particle swarm optimization (Reddy[6]), harmony search method (Lee & Geem[7]), and Big-Bang Big-Crunch (BB-BC) algorithm (O. Hasançebi,[8]), subset simulation (Li[9]), improved harmony search algorithm (Mahadavi[10]) and so on. All these deterministic optimizations aim to search the optimum solution under given constraints without consideration of uncertainties. So, while a deterministic optimization approach is unable to handle structural performances such as imprecise stresses and deflection etc. due to the presence of impreciseness, to get rid of such problem fuzzy (Zadeh,[11]), intuitionistic fuzzy (Atanassov,[12]) neutrosophic (Smarandache[20]) play great roles.

In IFS theory we usually consider degree of acceptance, and degree of rejection where as we consider only membership function in fuzzy set. Sarkar[13] optimize two bar truss design with imprecise load and stress in intuitionistic fuzzy environment calculating total integral values of triangular intuitionistic fuzzy number. Shu[14] applied triangular intuitionistic fuzzy number to fault tree analysis on printed board circuit assembly. P. Grzegorzewski *et.al*[15], H.B. Mitchell *et.al*[16], G. Nayagam *et.al*[17], H.M. Nehi *et.al*[18], S. Rezvani *et.al*[19] have been employed concept of intuitionistic fuzzy number in multi-attribute decision making(MADM) problem .So indeterminate information should be considered in decision making process. A few research work has been done on neutrosophic optimization in the field of structural optimization. So to deal with different impreciseness on load,stresses and deflection, we have been motivated to incorporate the concept of neutrosophic number in this problem, and have developed NSO algorithm to optimize the optimum design in imprecise environment.

In intuitionistic fuzzy number indeterminate information is partially lost ,as hesitant information is taken in consideration by default. So indeterminate information should be considered in decision making process. Smarandache[20] defined neutrosophic set that could handled indeterminate and inconsistent information. In neutrosophic sets indeterminacy is quantified explicitly with truth membership, indeterminacy membership and falsity membership function which are independent. Wang *et.al*[21] define single valued neutrosophic set which represents imprecise, incomplete, indeterminate, inconsistent information. Thus taking the universe as a real line we can develop the concept of single valued neutrosophic number as special case of neutrosophic sets. These numbers are able to express ill-known quantity with uncertain numerical value in decision making problem.

We define generalized triangular neutrosophic number and nearest interval approximation of this number. Then using parametric interval valued function for approximated interval number of neutrosophic number we solve welded beam design problem in neutrosophic environment. This paper develops optimization algorithm using max-min operator in neutrosophic environment to optimize the cost of welding, while the maximum shear stress in the weld group, maximum bending stress in the beam, and buckling load of the beam have been considered and deflection at the tip of a welded steel beam as constraints. Here parametric interval valued function of generalized triangular neutrosophic number have been considered for applied load , stress and deflection .The present study investigates computational algorithm for solving single-objective nonlinear programming problem by parametric neutrosophic optimization approach. The remainder of this paper is organized in the following manner. In section 2, we have discussed about single objective welded beam design. In section 3, we have discussed about neutrosophic set , neutrosophic number, α-cut and arithmetic operation on neutrosophic number. In section 4, we have discussed the solution procedure of single objective nonlinear programming problem by parametric neutrosophic non-linear programming technique. In section 5, we have discuss about solution of single objective welded beam optimization Problem by parametric neutrosophic optimization technique. In section 6, we have discussed about numerical solution of single-objective welded beam. Finally, we draw conclusions from the results in section 7.

2. Single-objective structural model

In the case of sizing optimization problems, the main motto of single objective structural optimization problem is to minimize objective function which are usually the cost of the structure or weight under certain behavioural constraints such as deflection ,load or stresses. The design variables are most frequently chosen from dimensions of the structure which are often height, length, depth, width e.t.c of the structure. Due to limitation of fabrications the design variables are not continuous rather discrete for belongingness of certain set. A discrete structural optimization problem can be formulated in the following form

$$Minimize\ \ C(X) \tag{1}$$

$$subject\ to$$

$$\sigma_i\left(X\right)\le\left[\sigma_i\left(X\right)\right], i=1,2,.....,m$$

$$X=\left\{x_1,x_2,.....x_n\right\}\in R^d$$

where $C(X)$ represents cost function, $\sigma_i\left(X\right)$ is the behavioural constraints and $\left[\sigma_i\left(X\right)\right]$ denotes the maximum allowable value , '*m*' and '*n*' are the number of constraints and design variables respectively. A given set of discrete value is expressed by R^d and in this paper objective function is taken as

$$C\left(X\right)=\sum_{t=1}^{T}c_t\prod_{n=1}^{m}x_n^{tn}$$

and constraint are chosen to be stress of structures as follows

$\sigma_i\left(A\right)\tilde{\le}^n\ \sigma_i^n$ with allowable tolerance σ_i^0 for $i=1,2,....,m$

and deflection of the structure as follows

$\delta\left(x\right)\tilde{\le}^n\ \delta_{\max}^n\left(x\right)$ with allowable tolerance $\delta_{\max}^0$

in neutrosophic environment. In this design formulation c_t is the coefficient of t^th^ term in cost function and x_n is the n^{th} design variable respectively. In constraint functions σ_i^n ,$\delta_{\max}^n\left(x\right)$ m, σ_i , σ_i^0 and $\delta_{\max}^0$ represent neutrosophic resources of stress, neutrosophic resources of deflection, the number of stress constraints i^{th}, stress, maximum allowable i^{th} stress and maximum allowable deflection respectively. $\tilde{\le}^n$ stands for inequality in neutrosophic sense.

3. Mathematical preliminaries

3.1. Single Valued Neutrosophic Set

A single valued neutrosophic set (SVNS) $\tilde{A}^n$ in the universe of discourse X is given $\tilde{A}^n=\left\{\left(x,T_{\tilde{A}^n}\left(x\right),I_{\tilde{A}^n}\left(x\right),F_{\tilde{A}^n}\left(x\right)\right)\middle|x\in X\right\}$ where $T_{\tilde{A}^n}:X\to\left[0,1\right]$, $I_{\tilde{A}^n}:X\to\left[0,1\right]$ and $F_{\tilde{A}^n}:X\to\left[0,1\right]$ with $0\le T_{\tilde{A}^n}\left(x\right)+I_{\tilde{A}^n}\left(x\right)+F_{\tilde{A}^n}\left(x\right)\le 3$ for all $x\in X$. The numbers $T_{\tilde{A}^n}\left(x\right)$,

$I_{\tilde{A}^n}(x),\ F_{\tilde{A}^n}(x),$ respectively represent the truth membership degree, indeterminacy membership degree, falsity membership degree of the element *x* to the set $\tilde{A}^n$.

3.2. $(\alpha,\beta,\gamma)-$cut of Single Valued Neutrosophic Set

$(\alpha,\beta,\gamma)-$cut of single valued neutrosophic set (SVNS) $\tilde{A}^n$, a crisp subset of $\Re$ is defined by $\tilde{A}^n_{\alpha,\beta,\gamma}=\left\{x \middle| T_{\tilde{A}^n}(x)\ge\alpha, I_{\tilde{A}^n}(x)\le\beta, F_{\tilde{A}^n}(x)\le\gamma\right\}$ where $\alpha,\beta,\gamma\in[0,1]$ and $0\le\alpha+\beta+\gamma\le 3.$

3.3. Normal Neutrosophic Set

A single valued neutrosophic set $\tilde{A}^n=\left\{\left(x,T_{\tilde{A}^n}(x),I_{\tilde{A}^n}(x),F_{\tilde{A}^n}(x)\right)\middle| x\in X\right\}$ is called neutrosophic normal if there exists at least three points $x_0,x_1,x_2\in X$ such that $T_{\tilde{A}^n}(x_0)=1$ $I_{\tilde{A}^n}(x_1)=1,\ I_{\tilde{A}^n}(x_2)=1.$

3.4. Convex Neutrosophic Set

A single valued neutrosophic set $\tilde{A}^n=\left\{\left(x,T_{\tilde{A}^n}(x),I_{\tilde{A}^n}(x),F_{\tilde{A}^n}(x)\right)\middle| x\in X\right\}$ is a subset of the real line called neut-convex if for all $x_1,x_2\in\Re$ and $\omega\in[0,1]$ the following conditions are satisfied.

1. $T_{\tilde{A}^n}\left\{\omega x_1+(1-\omega)x_2\right\}\ge\min\left\{T_{\tilde{A}^n}(x_1),T_{\tilde{A}^n}(x_2)\right\}$
2. $I_{\tilde{A}^n}\left\{\omega x_1+(1-\omega)x_2\right\}\le\max\left\{I_{\tilde{A}^n}(x_1),I_{\tilde{A}^n}(x_2)\right\}$
3. $F_{\tilde{A}^n}\left\{\omega x_1+(1-\omega)x_2\right\}\le\max\left\{F_{\tilde{A}^n}(x_1),F_{\tilde{A}^n}(x_2)\right\}$

i.e $\tilde{A}^n$ is neut-convex if its truth membership function is fuzzy convex, indeterminacy membership function is fuzzy concave and falsity membership function is fuzzy concave.

3.5. Single Valued Neutrosophic Number

A single valued neutrosophic set $\tilde{A}^n=\left\{\left(x,T_{\tilde{A}^n}(x),I_{\tilde{A}^n}(x),F_{\tilde{A}^n}(x)\right)\middle| x\in X\right\}$, subset of a real line, is called generalised neutrosophic number if

1. $\tilde{A}^n$ is neut- normal.
2. $\tilde{A}^n$ is neut- convex.
3. $T_{\tilde{A}^n}(x)$ is upper semi-continuous, $I_{\tilde{A}^n}(x)$ is lower semi continuous and $F_{\tilde{A}^n}(x)$ is lower semi continuous, and

4. the support of $\tilde{A}^n$, i.e $S\left(\tilde{A}^n\right)=\left\{x\in X: T_{\tilde{A}^n}>0, I_{\tilde{A}^n}<1, F_{\tilde{A}^n}<1\right\}$. is bounded.

Thus for any Single Valued Triangular Neutrosophic Number there exists nine numbers $a_1^T, a_2, a_3^T, b_1^I, b_2, b_3^I, c_1^F, c_2, c_3^F \in \Re$ such that $c_1^F \le b_1^I \le a_1^T \le c_2 \le b_2 \le a_2 \le a_3^T \le b_3^I \le c_3^F$ and six functions $T_{\tilde{A}^n}^L(x), I_{\tilde{A}^n}^L(x), F_{\tilde{A}^n}^L(x), T_{\tilde{A}^n}^R(x), I_{\tilde{A}^n}^R(x), F_{\tilde{A}^n}^R(x): \Re \to [0,1]$ represents truth, indeterminacy and falsity membership degree of $\tilde{A}^n$. The three non-decreasing functions $T_{\tilde{A}^n}^L(x), I_{\tilde{A}^n}^L(x), F_{\tilde{A}^n}^L(x)$ represents the left side of truth, indeterminacy and falsity membership functions of SVNN $\tilde{A}^n$ respectively. Similarly the three non-increasing functions $T_{\tilde{A}^n}^R(x), I_{\tilde{A}^n}^R(x), F_{\tilde{A}^n}^R(x)$ represent the right side of truth ,indeterminacy and falsity membership functions of SVNN $\tilde{A}^n$ respectively. The truth, indeterminacy and falsity membership functions of SVNN $\tilde{A}^n$ can be defined in the following way

$$T_{\tilde{A}^n}(x)=\begin{cases} T_{\tilde{A}^n}^L(x) & if\ a_1^T \le x \le a_2 \\ T_{\tilde{A}^n}^R(x) & if\ a_2 \le x \le a_3^T \\ 0 & otherwise \end{cases}; \quad I_{\tilde{A}^n}(x)=\begin{cases} I_{\tilde{A}^n}^L(x) & if\ b_1^I \le x \le b_2 \\ I_{\tilde{A}^n}^R(x) & if\ b_2 \le x \le b_3^I \\ 0 & otherwise \end{cases}$$

$$F_{\tilde{A}^n}(x)=\begin{cases} F_{\tilde{A}^n}^L(x) & if\ c_1^F \le x \le c_2 \\ F_{\tilde{A}^n}^R(x) & if\ c_2 \le x \le c_3^F \\ 0 & otherwise \end{cases}$$

The sum of three independent membership degree of SVNN $\tilde{A}^n$ lie between the interval $[0,3]$. i.e $0 \le T_{\tilde{A}^n}^R(x)+I_{\tilde{A}^n}^R(x)+F_{\tilde{A}^n}^R(x) \le 3 \ \ x \in \tilde{A}^n$.

3.6. Generalized Triangular Neutrosophic Number

A generalized single valued triangular Neutrosophic number $\tilde{A}^n$ with the set of parameters $c_1^F \le b_1^I \le a_1^T \le c_2 \le b_2 \le a_2 \le a_3^T \le b_3^I \le c_3^F$ denoted as $\tilde{A}^n=\left(\left(a_1^T, a_2, a_3^T; w_a\right),\left(b_1^I, b_2, b_3^I; \eta_a\right)\left(c_1^F, c_2, c_3^F; \tau_a\right)\right)$ is the set of real numbers $\Re$. The truth membership, indeterminacy membership and falsity membership functions of $\tilde{A}^n$ can be defined as follows

$$T_{\tilde{A}^n}=\begin{cases} w_a \dfrac{x-a_1^T}{a_2-a_1^T} & for\ a_1^T \le x \le a_2 \\ w_a & for\ x=a_2 \\ w_a \dfrac{a_3^T-x}{a_3^T-a_2} & for\ a_2 \le x \le a_3^T \\ 0 & otherwise \end{cases} \qquad I_{\tilde{A}^n}=\begin{cases} \eta_a \dfrac{x-b_1^I}{b_2-b_1^I} & for\ b_1^I \le x \le b_2 \\ \eta_a & for\ x=b_2 \\ \eta_a \dfrac{x-b_3^I}{b_3^I-b_2} & for\ b_2 \le x \le b_3^I \\ 0 & otherwise \end{cases}$$

$$F_{\tilde{A}^n}=\begin{cases}\lambda_a \dfrac{x-c_1^F}{c_2-c_1^F} & for\ c_1^F\le x\le c_2\\ \tau_a & for\ x=c_2\\ \lambda_a \dfrac{x-c_3^F}{c_3^F-c_2} & for\ c_2\le x\le c_3^F\\ 0 & otherwise\end{cases}$$

3.7. $(\alpha,\beta,\gamma)-$*cut Set of Single Valued Triangular Neutrosophic Number*

Let $\tilde{A}^n=\left(\left(a_1^T,a_2,a_3^T;w_a\right),\left(b_1^I,b_2,b_3^I;\eta_a\right)\left(c_1^F,c_2,c_3^F;\lambda_a\right)\right)$ be generalized single valued triangular Neutrosophic number. Then it is a crisp subset of $\Re$ and is defined by

$$A^n_{(\alpha,\beta,\gamma)}=\left\{x\,\middle|\,T_{\tilde{A}^n}(x)\ge\alpha, I_{\tilde{A}^n}(x)\le\beta, F_{\tilde{A}^n}(x)\le\gamma\right\}$$

$$=\left\{\left[L^\alpha(\tilde{A}),R^\alpha(\tilde{A})\right],\left[L^\beta(\tilde{A}),R^\beta(\tilde{A})\right],\left[L^\gamma(\tilde{A}),R^\gamma(\tilde{A})\right]\right\}$$

$$=\begin{Bmatrix}\left[a_1^T+\dfrac{\alpha}{w_a}\left(a_2-a_1^T\right),a_3^T-\dfrac{\alpha}{w_a}\left(a_3^T-a_2\right)\right],\\ \left[b_1^I+\dfrac{\beta}{\eta_a}\left(b_2-b_1^I\right),b_3^I+\dfrac{\beta}{\eta_a}\left(b_3^I-b_2\right)\right],\\ \left[c_1^F+\dfrac{\gamma}{\lambda_a}\left(c_2-c_1^F\right),c_3^F+\dfrac{\gamma}{\lambda_a}\left(c_3^F-c_2\right)\right]\end{Bmatrix}$$

4. Mathematical Analysis

4.1. Nearest Interval Approximation for Neutrosophic Number

Here we want to approximate an neutrosophic number $\tilde{A}^n=\left(\left(a_1^T,a_2,a_3^T;w_a\right),\left(b_1^I,b_2,b_3^I;\eta_a\right)\left(c_1^F,c_2,c_3^F;\lambda_a\right)\right)$ by a crisp model.

Let $\tilde{A}^n$ and $\tilde{B}^n$ be two neutrosophic number. Then the distance between them can be measured according to Euclidean matric as

$$\tilde{d}_E^2=\frac{1}{2}\int_0^1\left(T_{A_L}(\alpha)-T_{B_L}(\alpha)\right)^2d\alpha+\frac{1}{2}\int_0^1\left(T_{A_U}(\alpha)-T_{B_U}(\alpha)\right)^2d\alpha$$

$$+\frac{1}{2}\int_0^1\left(I_{A_L}(\alpha)-I_{B_L}(\alpha)\right)^2 d\alpha+\frac{1}{2}\int_0^1\left(I_{A_U}(\alpha)-I_{B_U}(\alpha)\right)^2 d\alpha$$

$$+\frac{1}{2}\int_0^1\left(F_{A_L}(\alpha)-F_{B_L}(\alpha)\right)^2 d\alpha+\frac{1}{2}\int_0^1\left(F_{A_U}(\alpha)-F_{B_U}(\alpha)\right)^2 d\alpha$$

Now we find a closed interval $\tilde{C}_{d_E}\left(\tilde{A}^i\right)=\left[C_L,C_U\right]$ which is nearest to $\tilde{A}^n$ with respect to the matric $\tilde{d}_E$. Again it is obvious that each real interval can also be considered as an intuitionistic fuzzy number with constant $\alpha-$cut $\left[C_L,C_U\right]$ for all $\alpha\in[0,1]$. Now we have to minimize $\tilde{d}_E\left(\tilde{A}^n,\tilde{C}_{d_E}\left(\tilde{A}^n\right)\right)$ with respect to C_L and C_U, that is to minimize $F_1(C_L,C_U)=\int_0^1\left(T_{A_L}(\alpha)-C_L\right)^2 d\alpha+\int_0^1\left(T_{A_U}(\alpha)-C_U\right)^2 d\alpha$

$$+\int_0^1\left(I_{A_L}(\alpha)-C_L\right)^2 d\alpha+\int_0^1\left(I_{A_U}(\alpha)-C_U\right)^2 d\alpha$$

$$+\int_0^1\left(F_{A_L}(\alpha)-C_L\right)^2 d\alpha+\int_0^1\left(F_{A_U}(\alpha)-C_U\right)^2 d\alpha$$

With respect to C_L and C_U. We define partial derivatives

$$\frac{\partial F_1(C_L,C_U)}{\partial C_L}=-2\int_0^1\left(T_{A_L}(\alpha)+I_{A_L}(\alpha)+F_{A_L}(\alpha)\right)d\alpha+6C_L$$

$$\frac{\partial F_1(C_L,C_U)}{\partial C_U}=-2\int_0^1\left(T_{A_U}(\alpha)+I_{A_U}(\alpha)+F_{A_U}(\alpha)\right)d\alpha+6C_U$$

And then we solve the system

$$\frac{\partial F_1(C_L,C_U)}{\partial C_L}=0,\frac{\partial F_1(C_L,C_U)}{\partial C_U}=0$$

The solution is

$$C_L=\int_0^1\frac{T_{A_L}(\alpha)+I_{A_L}(\alpha)+F_{A_L}(\alpha)}{3}d\alpha;\ C_U=\int_0^1\frac{T_{A_U}(\alpha)+I_{A_U}(\alpha)+F_{A_L}(\alpha)}{3}d\alpha$$

$$\det\begin{pmatrix}\frac{\partial^2 F_1(C_L,C_U)}{\partial C_L^2} & \frac{\partial^2 F_1(C_L,C_U)}{\partial C_L\partial C_U}\\ \frac{\partial^2 F_1(C_L,C_U)}{\partial C_U\partial C_L} & \frac{\partial^2 F_1(C_L,C_U)}{\partial C_U^2}\end{pmatrix}$$

Since

$=\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} = 36 > 0$ then $C_L \ C_U$ mentioned above minimize $F_1\left(C_L, C_U\right)$. The nearest interval of the intuitionistic fuzzy number $\tilde{A}^i$ with respect to the matric $\tilde{d}_E$ is

$$\tilde{C}_{d_E}\left(\tilde{A}^n\right) = \left[\int_0^1 \frac{T_{A_L}(\alpha) + I_{A_L}(\alpha) + F_{A_L}(\alpha)}{3} d\alpha, \int_0^1 \frac{T_{A_U}(\alpha) + I_{A_U}(\alpha) + F_{A_U}(\alpha)}{3} d\alpha\right]$$

$$= \left[\frac{a_1^T + b_1^I + c_1^F}{3} + \frac{a_2 - a_1^T}{6w_a} + \frac{b_2 - b_1^I}{6\eta_a} + \frac{c_2 - c_1^F}{6\lambda_a}, \frac{a_3^T + b_3^I + c_3^F}{3} + \frac{a_2 - a_3^T}{6w_a} + \frac{b_3^I - b_2}{6\eta_a} + \frac{c_3^F - c_2}{6\lambda_a}\right]$$

4.2. Parametric Interval valued Function

If $[m,n]$ be an interval with $m,n>0$ we can express an interval number by a function. The parametric interval-valued function for the interval $[m,n]$ can be taken as $g(s) = m^{1-s}n^s$ for $s \in [0,1]$ which is strictly monotone continuous function and its inverse exists. Let ψ be the inverse of $g(s)$ then $s = \frac{\log\psi - \log m}{\log n - \log m}$.

4.3. Formulation of Neutrosophic Programming with imprecise coefficient in parametric form

A multi-objective intuitionistic fuzzy non-linear programming problem with imprecise co-efficient can be formulated as

$$\textit{Minimize } \tilde{f}(x) = \sum_{t=1}^{T} \xi_t \tilde{c}_t^n \prod_{j=1}^{n} x_j^{a_{tj}}$$

Such that $\tilde{f}_i(x) = \sum_{t=1}^{T_i} \xi_{it} \tilde{c}_{it}^n \prod_{j=1}^{n} x_j^{a_{itj}} \le \xi_i \tilde{b}_i^n$ for $i = 1,2,....,m$

$$x_j > 0 \quad j = 1,2,....,n$$

Here ξ_t, ξ_{it}, ξ_i are the signum function used to indicate sign of term in the equation. $\tilde{c}_t > 0$, $\tilde{c}_{it} > 0$. a_{tj}, a_{itj} are real numbers for all i,t,j.

Here $\tilde{c}_t^{\ n} = \left(\left(c_t^{1T}, c_t^{2T}, c_t^{3T}; w_{c_t}\right), \left(c_t^{1I}, c_t^{2I}, c_t^{3I}; \eta_{c_t}\right)\left(c_t^{1F}, c_t^{2F}, c_t^{3F}; \tau_{c_t}\right)\right)$

$\tilde{c}_{it}^{\ n} = \left(\left(c_{it}^{1T}, c_{it}^{2T}, c_{it}^{3T}; w_{c_{it}}\right), \left(c_{it}^{1I}, c_{it}^{2I}, c_{it}^{3I}; \eta_{c_{it}}\right)\left(c_{it}^{1F}, c_{it}^{2F}, c_{it}^{3F}; \tau_{c_{it}}\right)\right)$

$\tilde{b}_i^n = \left(\left(b_i^{1T}, b_i^{2T}, b_i^{3T}; w_{b_i}\right), \left(b_i^{1I}, b_i^{2I}, b_i^{3I}; \eta_{b_i}\right)\left(b_i^{1F}, b_i^{2F}, b_i^{3F}; \tau_{b_i}\right)\right)$ for neutrosophic number coefficient.

Using nearest interval approximation method for both fuzzy and intuitionistic fuzzy number, we transform all the triangular intuitionistic fuzzy number into interval number i.e $\left[c_t^L, c_t^U\right]$, $\left[c_{it}^L, c_{it}^U\right]$, and $\left[b_i^L, b_i^U\right]$
Now the intuitionistic multi-objective programming with imprecise parameter is of the following form

$$Minimize\ \ \hat{f}(x) = \sum_{t=1}^{T} \xi_{k_0 t} \hat{c}_t \prod_{j=1}^{n} x_j^{a_{tj}}$$

Such that $\hat{f}_i(x) = \sum_{t=1}^{T_i} \xi_{it} \hat{c}_{it} \prod_{j=1}^{n} x_j^{a_{itj}} \le \sigma_i \hat{b}_i$ for $i = 1, 2, ..., m$

$$x_j > 0 \quad j = 1, 2, ..., n$$

Here ξ_t, ξ_{it}, ξ_i are the signum function used to indicate sign of term in the equation. $\hat{c}_t > 0$, $\hat{c}_{it} > 0$; $\hat{b}_i > 0$ denote the interval component i.e $\hat{c}_t = \left[c_t^L, c_t^U\right]$, $\hat{c}_{it} = \left[c_{it}^L, c_{it}^U\right]$, and $\hat{b}_i = \left[b_i^L, b_i^U\right]$ and a_{tj}, a_{itj} are real numbers for all i, t, j.
Using parametric interval valued function the above problem transform into

$$Minimize\ \ f(x;s) = \sum_{t=1}^{T} \xi_t \left(c_t^L\right)^{1-s} \left(c_t^U\right)^s \prod_{j=1}^{n} x_j^{a_{tj}}$$

Such that $f_i(x;s) = \sum_{t=1}^{T_i} \xi_{it} \left(c_{it}^L\right)^{1-s} \left(c_{it}^U\right)^s \prod_{j=1}^{n} x_j^{a_{itj}} \le \xi_i \left(b_i^L\right)^{1-s} \left(b_i^U\right)^s$ for $i = 1, 2, ..., m$

$$x_j > 0 \quad j = 1, 2, ..., n \ \ s \in [0,1]$$

Here ξ_t, ξ_{it}, ξ_i are the signum function used to indicate sign of term in the equation.
This is a parametric single objective non-linear programming problem and can be solved by intuitionistic fuzzy optimization technique.

4.5. NSO to solve Parametric Single-Objective Non-linear Programming Problem (PSONLP)

Let us consider a single-objective nonlinear optimization problem as

$$Minimize\quad f(x;s) \tag{2}$$

$$g_j(x;s) \le b_j(s) \qquad j = 1, 2, ..., m$$

$$x \ge 0 \ \ s \in [0,1]$$

Usually constraints goals are considered as fixed quantity. But in real life problem, the constraint goal can not be always exact. So we can consider the constraint goal for less than type constraints at least $b_j(s)$ and it may possible to extend to $b_j(s)+b_j^0(s)$. This fact seems to take the constraint goal as a neutrosophic fuzzy set and which will be more realistic descriptions than others. Then the NLP becomes NSO problem with neutrosophic resources, which can be described as follows

$$Minimize \quad f(x;s) \tag{3}$$

$$g_j(x;s) \tilde{\leq}^n \tilde{b}_j^n(s) \qquad j=1,2,....,m$$

$$x \geq 0 \quad s \in [0,1]$$

To solve the NSO (3), following warner's (1987) and Angelov (1995) we are presenting a solution procedure for single-objective NSO problem (3) as follows

Step-1: Following warner's approach solve the single objective non-linear programming problem without tolerance in constraints (i.e $g_j(x;s) \leq b_j(s)$), with tolerance of acceptance in constraints (i.e $g_j(x;s) \leq b_j(s)+b_j^0(s)$) by appropriate non-linear programming technique Here they are

Sub-problem-1

$$Minimize \quad f(x;s) \tag{4}$$

$$g_j(x;s) \leq b_j(s) \qquad j=1,2,....,m$$

$$x \geq 0 \quad s \in [0,1]$$

Sub-problem-2

$$Minimize \quad f(x;s) \tag{5}$$

$$g_j(x;s) \leq b_j(s)+b_j^0(s), \qquad j=1,2,....,m$$

$$x \geq 0 \quad s \in [0,1]$$

we may get optimal solutions $x^* = x^1,\ f(x^*) = f(x^1)$ and $x^* = x^2,\ f(x^*) = f(x^2)$ for sub-problem 1 and 2 respectively.

Step-2: From the result of step 1 we now find the lower bound and upper bound of objective functions. If $U^T_{f(x;s)}, U^I_{f(x;s)}, U^F_{f(x;s)}$ be the upper bounds of truth, indeterminacy, falsity function for the objective respectively and $L^T_{f(x;s)}, L^I_{f(x;s)}, L^F_{f(x;s)}$ be the lower bound of truth, indeterminacy, falsity membership functions of objective for particular values of $s \in [0,1]$ respectively then

$$U^T_{f(x;s)} = \max\left\{ f(x^1;s), f(x^2;s) \right\}, L^T_{f(x;s)} = \min\left\{ f(x^1;s), f(x^2;s) \right\},$$

$$U^F_{f(x;s)} = U^T_{f(x;s)}, L^F_{f(x;s)} = L^T_{f(x;s)} + t\left(U^T_{f(x;s)} - L^T_{f(x;s)}\right)$$

$$L^I_{f(x;s)} = L^T_{f(x;s)}, U^I_{f(x;s)} = L^T_{f(x;s)} + q\left(U^T_{f(x;s)} - L^T_{f(x;s)}\right)$$

Here t,q are predetermined real numbers in (0,1)

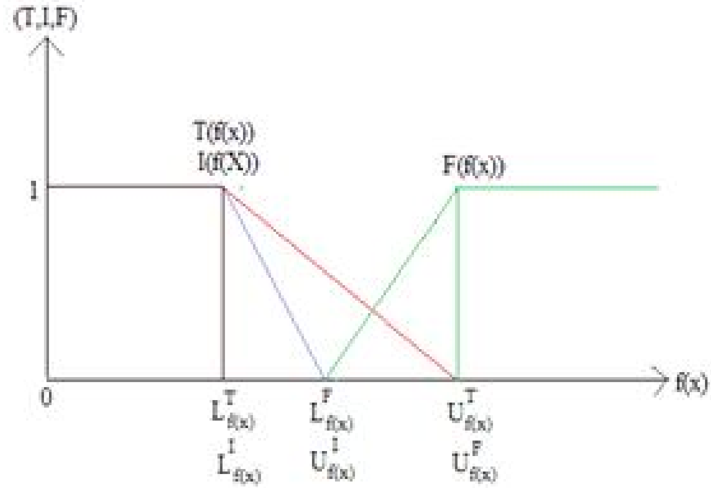

Fig.1. Rough Sketch of Lower and Upper bounds of Truth, Indeterminacy and Falsity Membership Functions

Step-3: In this step we calculate linear membership for truth, indeterminacy and falsity membership functions of objective as follows

$$T_{f(x;s)}\left(f\left(x;s\right)\right) = \begin{cases} 1 & if \quad f\left(x;s\right) \le L^T_{f(x;s)} \\ \left(\dfrac{U^T_{f(x;s)} - f\left(x;s\right)}{U^T_{f(x;s)} - L^T_{f(x;s)}}\right) & if \quad L^T_{f(x;s)} \le f\left(x;s\right) \le U^T_{f(x;s)} \\ 0 & if \quad f\left(x;s\right) \ge U^T_{f(x;s)} \end{cases}$$

$$I_{f(x;s)}\left(f\left(x;s\right)\right) = \begin{cases} 1 & if \quad f\left(x;s\right) \le L^I_{f(x;s)} \\ \left(\dfrac{U^I_{f(x;s)} - f\left(x;s\right)}{U^I_{f(x;s)} - L^I_{f(x;s)}}\right) & if \quad L^I_{f(x;s)} \le f\left(x;s\right) \le U^I_{f(x;s)} \\ 0 & if \quad f\left(x;s\right) \ge U^I_{f(x;s)} \end{cases}$$

$$F_{f(x;s)}\left(f(x;s)\right)=\begin{cases}0 & if\ \ f(x;s)\le L^F_{f(x;s)}\\ \dfrac{f(x;s)-L^F_{f(x;s)}}{U^F_{f(x;s)}-L^F_{f(x;s)}} & if\ \ L^F_{f(x;s)}\le f(x;s)\le U^F_{f(x;s)}\\ 1 & if\ \ f(x;s)\ge U^F_{f(x;s)}\end{cases}$$

Step-4: In this step using linear function for truth,indeterminacy and falsity membership functions, we may calculate membership function for constraints as follows

$$T_{g_j(x;s)}\left(g_j(x;s)\right)=\begin{cases}1 & if\ \ g_j(x;s)\le b_j\\ \left(\dfrac{b_j(s)+b_j^0(s)-g_j(x;s)}{b_j^0}\right) & if\ \ b_j(s)\le g_j(x;s)\le b_j(s)+b_j^0(s)\\ 0 & if\ \ g_j(x;s)\ge b_j^0(s)\end{cases}$$

$$I_{g_j(x;s)}\left(g_j(x;s)\right)=\begin{cases}1 & if\ \ g_j(x;s)\le b_j(s)\\ \dfrac{\left(b_j(s)+\xi_{g_j(x;s)}\right)-g_j(x;s)}{\xi_{g_j(x;s)}} & if\ \ b_j(s)\le g_j(x;s)\le b_j(s)+\xi_{g_j(x;s)}\\ 0 & if\ \ g_j(x;s)\ge b_j(s)+\xi_{g_j(x;s)}\end{cases}$$

$$F_{g_j(x;s)}\left(g_j(x;s)\right)=\begin{cases}0 & if\ \ g_j(x;s)\le b_j(s)+\varepsilon_{g_j(x;s)}\\ \dfrac{g_j(x;s)-b_j(s)-\varepsilon_{g_j(x;s)}}{b_j^0(s)-\varepsilon_{g_j(x;s)}} & if\ \ b_j(s)+\varepsilon_{g_j(x;s)}\le g_j(x;s)\le b_j(s)+b_j^0(s)\\ 1 & if\ \ g_j(x,s)\ge b_j(s)+b_j^0(s)\end{cases}$$

where and for $j=1,2,.....,m\quad t,q\in(0,1)$.

Step-5: Now using NSO for single objective optimization technique the optimization problem (2) can be formulated as

Model-I

$$Maximize\ (\alpha+\gamma-\beta)$$

Suh that

$$T_{f(x;s)}(x;s)\ge\alpha;\ \ T_{g_j}(x;s)\ge\alpha; \tag{6}$$

$$I_{f(x,s)}(x;s) \geq \gamma;\ \ I_{g_j}(x;s) \geq \gamma;$$

$$F_{f(x;s)}(x;s) \leq \beta;\ F_{g_j}(x;s) \leq \beta;$$

$$\alpha + \beta + \gamma \leq 3;\ \alpha \geq \beta; \alpha \geq \gamma;$$

Now the above problem (6) can be simplified to following crisp linear programming problem for linear membership function as

$$Maximize\ \ (\alpha + \gamma - \beta) \tag{7}$$

$$such\ that\ \ f(x;s) + \left(U^T - L^T\right).\alpha \leq U^T;$$

$$f(x;s) + \left(U^I_{f(x;s)} - L^I_{f(x;s)}\right).\gamma \geq U^I_{f(x;s)};$$

$$f(x;s) - \left(U^F_{f(x;s)} - L^F_{f(x;s)}\right).\beta \leq L^F_{f(x;s)}$$

$$\alpha + \beta + \gamma \leq 3;\ \ \alpha \geq \beta; \alpha \geq \gamma;$$

$$g_j(x;s) + \left(U^T - L^T\right).\alpha \leq U^T;$$

$$g_j(x;s) + \left(U^I_{g_j(x;s)} - L^I_{g_j(x;s)}\right).\gamma \geq U^I_{g_j(x;s)};$$

$$g_j(x;s) - \left(U^F_{g_j(x;s)} - L^F_{g_j(x;s)}\right).\beta \leq L^F_{g_j(x;s)}$$

$$\alpha, \beta, \gamma \in [0,1]\quad s \in [0,1]\quad j = 1,2,....,m$$

This crisp nonlinear programming problems (7) can be solved by appropriate mathematical algorithm.

5. Solution of Single-Objective Welded Beam Design(SOWBD) using NSO Technique

The parametric Welded beam design problem can be formulated as

$$Minimize\ \ C(X;s) \tag{8}$$

$$subject\ to\ \sigma_i(X;s) \leq [\sigma_i(s)], i = 1,2,.....,m$$

$$X_j \in R^d,\quad j = 1,2,.....,n$$

$$X > 0; s \in [0,1]$$

where $C(X;s)$ represents cost function, $\sigma_i(X;s)$ is the behavioural constraints and $[\sigma_i(X;s)]$ denotes the maximum allowable value , '*m*' and '*n*' are the number of constraints and design variables respectively. A given set of discrete value is expressed by R^d and in this-paper objective function is taken as

$$C(X;s)=\sum_{t=1}^{T}c_t(s)\prod_{n=1}^{m}x_n^{tn}$$

and constraint are chosen to be stress of structures as follows

$\sigma_i(X;s)\underset{\sim}{\le}^n \sigma_i(s)$ with allowable tolerance $\sigma_i^0(s)$ for $i=1,2,....,m$

And deflection of the structure as follows

$\delta(X;s)\underset{\sim}{\le}^n \delta_{\max}(s)$ with allowable tolerance $\delta_{\max}^0(s)$

Where c_t is the cost coefficient of t[th] side and x_n is the n^{th} design variable respectively, m is the number of structural element, σ_i and $\sigma_i^0(s)$ $\delta_{\max}^0(s)$ are the i^{th} stress, allowable stress and allowable deflection respectively. $\underset{\sim}{\le}^n$ represents less than or equal to in neutrosophic sense.

To solve the SOWBP (1), step 1 of 4 is used and let $U_{C(X;s)}^T, U_{C(X;s)}^I, U_{C(X;s)}^F$ be the upper bounds of truth, indeterminacy, falsity function for the objective respectively and $L_{C(X;s)}^T, L_{C(X;s)}^I, L_{C(X;s)}^F$ be the lower bound of truth, indeterminacy, falsity membership functions of objective respectively then

$$U_{C(X;s)}^T=\max\left\{C\left(X^1;s\right),C\left(X^2;s\right)\right\}, L_{f(x;s)}^T=\min\left\{C\left(X^1;s\right),C\left(X^2;s\right)\right\},$$

$$U_{C(X;s)}^F=U_{C(X;s)}^T, L_{C(X;s)}^F=L_{C(X;s)}^T+\varepsilon_{C(X;s)} \quad where\ 0<\varepsilon_{C(X;s)}<\left(U_{C(X;s)}^T-L_{C(X;s)}^T\right)$$

$$L_{C(X;s)}^I=L_{C(X;s)}^T, U_{C(X;s)}^I=L_{C(X;s)}^T+\xi_{C(X;s)} \quad where\ 0<\xi_{C(X;s)}<\left(U_{C(X;s)}^T-L_{C(X;s)}^T\right)$$

Let the linear membership function for objective be

$$T_{C(X;s)}\left(C(X;s)\right)=\begin{cases}1 & if\ C(X;s)\le L_{C(X;s)}^T\\ \left(\dfrac{U_{C(X;s)}^T-C(X;s)}{U_{C(X;s)}^T-L_{C(X;s)}^T}\right) & if\ L_{C(X;s)}^T\le C(X;s)\le U_{C(X;s)}^T\\ 0 & if\ C(X;s)\ge U_{C(X;s)}^T\end{cases}$$

$$I_{C(X;s)}\left(C(X;s)\right)=\begin{cases}1 & if\ C(X;s)\le L_{C(X;s)}^I\\ \left(\dfrac{U_{C(X;s)}^I-C(X;s)}{U_{C(X;s)}^I-L_{C(X;s)}^I}\right) & if\ L_{C(X;s)}^I\le C(X;s)\le U_{C(X;s)}^I\\ 0 & if\ C(X;s)\ge U_{C(X;s)}^I\end{cases}$$

$$F_{C(X;s)}\left(C(X;s)\right)=\begin{cases}0 & if\ C(X;s)\le L^F_{C(X;s)}\\ \dfrac{C(X;s)-L^F_{C(X;s)}}{U^F_{C(X;s)}-L^F_{C(X;s)}} & if\ L^F_{C(X;s)}\le C(X;s)\le U^F_{C(X;s)}\\ 1 & if\ C(X;s)\ge U^F_{C(X;s)}\end{cases}$$

and constraints be

$$T_{g_j(x;s)}\left(g_j(x;s)\right)=\begin{cases}1 & if\ g_j(x;s)\le b_j(s)\\ \left(\dfrac{b_j(s)+b_j^0(s)-g_j(x;s)}{b_j^0(s)}\right) & if\ b_j(s)\le g_j(x;s)\le b_j(s)+b_j^0(s)\\ 0 & if\ g_j(x;s)\ge b_j^0(s)\end{cases}$$

$$I_{g_j(x;s)}\left(g_j(x;s)\right)=\begin{cases}1 & if\ g_j(x;s)\le b_j(s)\\ \dfrac{\left(b_j(s)+\xi_{g_j(x;s)}\right)-g_j(x;s)}{\xi_{g_j(x;s)}} & if\ b_j(s)\le g_j(x;s)\le b_j(s)+\xi_{g_j(x;s)}\\ 0 & if\ g_j(x;s)\ge b_j(s)+\xi_{g_j(x;s)}\end{cases}$$

$$F_{g_j(x;s)}\left(g_j(x;s)\right)=\begin{cases}0 & if\ g_j(x;s)\le b_j(s)+\varepsilon_{g_j(x;s)}\\ \dfrac{g_j(x;s)-b_j(s)-\varepsilon_{g_j(x;s)}}{b_j^0(s)-\varepsilon_{g_j(x;s)}} & if\ b_j(s)+\varepsilon_{g_j(x;s)}\le g_j(x;s)\le b_j(s)+b_j^0(s)\\ 1 & if\ g_j(x;s)\ge b_j(s)+b_j^0(s)\end{cases}$$

where and for $j=1,2,.....,m \quad 0<\varepsilon_{g_j(x;s)},\xi_{g_j(x;s)}<b_j^0$

where and for $g_j(X;s)=\sigma_j(X;s)\ or\ \delta_j(X;s) or\ \tau_j(X;s), 0<\varepsilon_{g_j(X;s)}<b_j^0(s)$

then parametric NSO problem can be formulated as [22]

$$Maximize\ \ (\alpha+\gamma-\beta) \tag{9}$$

$$such\ that\ C(X;s)+\left(U^T_{C(X;s)}-L^T_{C(X;s)}\right).\alpha\le U^T_{C(X;s)};$$

$$C(X;s)+\left(U^I_{C(X;s)}-L^I_{C(X;s)}\right).\gamma\ge U^I_{C(X;s)};$$

$$C(X;s)-\left(U^F_{C(X;s)}-L^F_{C(X;s)}\right).\beta \le L^F_{C(X;s)};$$

$$g_j(x;s)+\left(U^T_{g_j(x;s)}-L^T_{g_j(x;s)}\right).\alpha \le U^T_{g_j(x;s)};$$

$$g_j(x;s)+\left(U^I_{g_j(x;s)}-L^I_{g_j(x;s)}\right).\gamma \ge U^I_{g_j(x;s)};$$

$$g_j(x;s)-\left(U^F_{g_j(x;s)}-L^F_{g_j(x;s)}\right).\beta \le L^F_{g_j(x;s)};$$

$$\alpha+\beta+\gamma \le 3; \alpha \ge \beta; \alpha \ge \gamma; \alpha,\beta,\gamma \in [0,1];$$

$$x \ge 0, s \in [0,1]$$

where $g_j(X;s)=\sigma_j(X;s)$ or $\delta_j(X;s)$ or $\tau_j(X;s), 0<\varepsilon_{g_j(X;s)}<b_j^0(s)$

All these crisp nonlinear programming problems (10) can be solved by appropriate mathematical algorithm.

6. Numerical Illustration

A welded beam (Ragsdell and Philips 1976, Fig. 2) has to be designed at minimum cost whose constraints are shear stress in weld (τ), bending stress in the beam (σ), buckling load on the bar (P), and deflection of the beam (δ). The design variables are $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} h \\ l \\ t \\ b \end{bmatrix}$ where h is the the weld size, l is the length of the weld, t is the depth of the welded beam, b is the width of the welded beam.

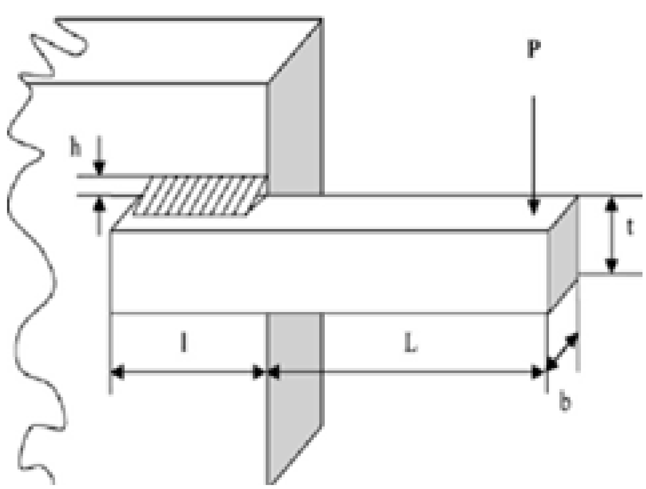

Fig. 2. Design of the welded beam

The single-objective optimization problem can be stated as follows

$$Minimize\ C(X) \equiv 1.10471 x_1^2 x_2 + 0.04811(14 + x_2) x_3 x_4 \tag{10}$$

Such that

$$g_1(x) \equiv \tau(x) - \tau_{\max} \le 0;$$

$$g_2(x) \equiv \sigma(x) - \sigma_{\max} \le 0;$$

$$g_3(x) \equiv x_1 - x_4 \le 0;$$

$$g_4(x) \equiv 0.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14 + x_2) - 5 \le 0;$$

$$g_5(x) \equiv 0.125 - x_1 \le 0;$$

$$g_6(x) \equiv \delta(x) - \delta_{\max} \le 0;$$

$$g_7(x) \equiv P - P_C(x) \le 0;$$

$$0.1 \le x_1, x_4 \le 2.0$$

$$0.1 \le x_2, x_3 \le 2.0$$

where $\tau(x) = \sqrt{\tau_1^2 + 2\tau_1\tau_2 \frac{x_2}{2R} + \tau_2^2}$; $\tau_1 = \frac{P}{\sqrt{2} x_1 x_2}$; $\tau_2 = \frac{MR}{J}$; $M = P\left(L + \frac{x_2}{2}\right)$;

$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$; $J = \left\{\frac{x_1 x_2}{\sqrt{2}}\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}$; $\sigma(x) = \frac{6PL}{x_4 x_3^2}$; $\delta(x) = \frac{4PL^3}{E x_4 x_3^2}$;

$P_C(x) = \frac{4.013\sqrt{E G x_3^6 x_4^6 / 36}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$; $P =$ Force on beam ; $L =$ Beam length beyond weld; $x_1 =$ Height of the welded beam; $x_2 =$ Length of the welded beam; $x_3 =$ Depth of the welded beam; $x_4 =$ Width of the welded beam; $\tau(x) =$ Design shear stress; $\sigma(x) =$ Design normal stress for beam material; $M =$ Moment of P about the centre of gravity of the weld, $J =$ Polar moment of inertia of weld group; $G =$ Shearing modulus of Beam Material; $E =$ Young modulus; $\tau_{\max} =$ Design Stress of the weld; $\sigma_{\max} =$ Design normal stress for the beam material; $\delta_{\max} =$ Maximum deflection; $\tau_1 =$ Primary stress on weld throat. $\tau_2 =$ Secondary torsional stress on weld. Input data are given in table 1and 2.

Table 1. Input data for crisp model (10)

Applied load P	Beam length beyond weld $L\ (in)$	Young Modulus $E\ (psi)$	Value of $G\ (psi)$
$\tilde{6000} \equiv \begin{pmatrix} (5580,6000,6100;w_p) \\ (5575,5590,6110;\eta_p) \\ (5570,5585,6120;\lambda_p) \end{pmatrix}$	14	3×10^6	12×10^6

Table 2. Input data for crisp model (10)

Maximum allowable shear stress $\tau_{\max}$ (psi)	Maximum allowable deflection $\delta_{\max}$ (in)	Maximum allowable normal stress $\sigma_{\max}$ (psi)
$\tilde{13550} \equiv \begin{pmatrix} (13520,13550,13580;w_\tau) \\ (13510,13540,13570;\eta_\tau) \\ (13500,13530,13560;\lambda_\tau) \end{pmatrix}$	$0.2\tilde{5} \equiv \begin{pmatrix} (0.22,0.25,0.26;w_\delta) \\ (0.21,0.24,0.27;\eta_\delta) \\ (0.20,0.23,0.28;\lambda_\delta) \end{pmatrix}$	$\tilde{3000} \equiv \begin{pmatrix} (2980,3000,3030;w_\sigma) \\ (2975,2990,3020;\eta_\sigma) \\ (2970,2985,3010;\lambda_\sigma) \end{pmatrix}$
Maximum allowable value $\tilde{13600} \equiv \begin{pmatrix} (13580,13600,13610;w_\tau^1) \\ (13575,13590,13615;\eta_\tau^1) \\ (13570,13585,136120;\lambda_\tau^1) \end{pmatrix}$	Maximum allowable value $0.2\tilde{6} \equiv \begin{pmatrix} (0.23,0.26,0.27;w_\delta^1) \\ (0.22,0.25,0.28;\eta_\delta^1) \\ (0.21,0.26,0.29;\lambda_\delta^1) \end{pmatrix}$	Maximum allowable value $\tilde{3100} \equiv \begin{pmatrix} (3070,3100,3130;w_\sigma^1) \\ (3060,3090,3120;\eta_\sigma^1) \\ (3050,3080,3110;\lambda_\sigma^1) \end{pmatrix}$

where $w_p, w_\sigma, w_\delta, w_\tau$ and $w_p, w_\sigma^1, w_\delta^1, w_\tau^1$ are degree of truth membership or aspiration level and maximum degree of truth membership or aspiration level; $\eta_p, \eta_\sigma, \eta_\delta, \eta_\tau$; $\eta_p, \eta_\sigma^1, \eta_\delta^1, \eta_\tau^1$ are degree of indeterminacy and maximum degree of indeterminacy and $\lambda_p, \lambda_\sigma, \lambda_\delta, \lambda_\tau$ and $\lambda_p, \lambda_\sigma^1, \lambda_\delta^1, \lambda_\tau^1$ are degree of falsity and maximum degree of falsity or desperation level of applied load, normal stress ,deflection and allowable shear stress respectively .

Now parameterized value of interval valued function can be calculated as

$$\hat{P} = \left(\left(5575 + \frac{70}{w_a} + \frac{2.5}{\eta_a} + \frac{2.5}{\tau_a} \right)^{1-s} \left(6110 - \frac{16.67}{w_a} + \frac{86.67}{\eta_a} + \frac{89.17}{\tau_a} \right)^{s} \right);$$

$$\hat{\tau}^{\max} = \left(\left(13510 + \frac{5}{w_a} + \frac{5}{\eta_a} + \frac{5}{\tau_a} \right)^{1-s} \left(13570 - \frac{1.67}{w_a} + \frac{5}{\eta_a} + \frac{5}{\tau_a} \right)^{s} \right);$$

Allowable value of $\hat{\tau}^{\max}$

$$\hat{\tau}_1^{\max} = \left(\left(13575 + \frac{3.33}{w_a} + \frac{2.5}{\eta_a} + \frac{2.5}{\tau_a}\right)^{1-s}\left(13615 - \frac{1.67}{w_a} + \frac{4.17}{\eta_a} + \frac{5.83}{\tau_a}\right)^{s}\right);$$

$$\hat{\delta}^{\max} = \left(\left(0.21 + \frac{0.005}{w_a} + \frac{0.005}{\eta_a} + \frac{0.005}{\tau_a}\right)^{1-s}\left(0.27 - \frac{0.001}{w_a} + \frac{0.005}{\eta_a} + \frac{0.008}{\tau_a}\right)^{s}\right);$$

Allowable value of $\hat{\delta}^{\max}$

$$\hat{\delta}_1^{\max} = \left(\left(0.22 + \frac{0.005}{w_a} + \frac{0.005}{\eta_a} + \frac{0.005}{\tau_a}\right)^{1-s}\left(0.28 - \frac{0.001}{w_a} + \frac{0.005}{\eta_a} + \frac{0.008}{\tau_a}\right)^{s}\right);$$

$$\hat{\sigma}^{\max} = \left(\left(2975 + \frac{3.33}{w_a} + \frac{2.5}{\eta_a} + \frac{2.5}{\tau_a}\right)^{1-s}\left(3020 - \frac{1.67}{w_a} + \frac{5}{\eta_a} + \frac{7.5}{\tau_a}\right)^{s}\right);$$

Allowable value of $\hat{\sigma}^{\max}$

$$\hat{\sigma}_1^{\max} = \left(\left(3060 + \frac{5}{w_a} + \frac{5}{\eta_a} + \frac{5}{\tau_a}\right)^{1-s}\left(3120 - \frac{1.67}{w_a} + \frac{5}{\eta_a} + \frac{8.83}{\tau_a}\right)^{s}\right);$$

Table 3. The Upper and lower value of objective for different values of w pessimistic value of s

The pessimistic value of s=0.2			
Aspiration level, uncertainty level and desperation level $w_p = w_\sigma = w_\delta = w_\tau = w_p = w_\sigma^1 = w_\delta^1 = w_\tau^1 = w$ $\eta_p = \eta_\sigma = \eta_\delta = \eta_\tau = \eta_\sigma^1 = \eta_\delta^1 = \eta_\tau^1 = \eta$ $\lambda_p = \lambda_\sigma = \lambda_\delta = \lambda_\tau = \lambda_\sigma^1 = \lambda_\delta^1 = \lambda_\tau^1 = \lambda$	$w = \eta = \lambda = 0.3$	$w = \eta = \lambda = 0.5$	$w = \eta = \lambda = 0.7$
Upper and lower value of objective	$L^T_{C(X)} = 0.1419847$, $U^T_{C(X)} = 0.1425069$	$L^T_{C(X)} = 0.1387723$, $U^T_{C(X)} = 0.1393634$	$L^T_{C(X)} = 0.1374016$, $U^T_{C(X)} = 0.1380209$

Table 4. The Upper and lower value of objective for different values of w, moderate value of value of s

The pessimistic value of s=0.5			
Aspiration level, uncertainty level and desperation level $w_p = w_\sigma = w_\delta = w_\tau = w_p = w_\sigma^1 = w_\delta^1 = w_\tau^1 = w$ $\eta_p = \eta_\sigma = \eta_\delta = \eta_\tau = \eta_\sigma^1 = \eta_\delta^1 = \eta_\tau^1 = \eta$ $\lambda_p = \lambda_\sigma = \lambda_\delta = \lambda_\tau = \lambda_\sigma^1 = \lambda_\delta^1 = \lambda_\tau^1 = \lambda$	$w = \eta = \lambda = 0.3$	$w = \eta = \lambda = 0.5$	$w = \eta = \lambda = 0.7$
Upper and lower value of objective	$L_{C(x)}^T = 0.1485833$, $U_{C(x)}^T = 0.1491453$	$L_{C(x)}^T = 0.1444032$, $U_{C(x)}^T = 0.1450005$	$L_{C(x)}^T = 0.1426218$, $U_{C(x)}^T = 0.1432331$

Table 5. The Upper and lower value of objective for different values of w optimistic value of s

The pessimistic value of s=0.8			
Aspiration level, uncertainty level and desperation level $w_p = w_\sigma = w_\delta = w_\tau = w_p = w_\sigma^1 = w_\delta^1 = w_\tau^1 = w$ $\eta_p = \eta_\sigma = \eta_\delta = \eta_\tau = \eta_\sigma^1 = \eta_\delta^1 = \eta_\tau^1 = \eta$ $\lambda_p = \lambda_\sigma = \lambda_\delta = \lambda_\tau = \lambda_\sigma^1 = \lambda_\delta^1 = \lambda_\tau^1 = \lambda$	$w = \eta = \lambda = 0.3$	$w = \eta = \lambda = 0.5$	$w = \eta = \lambda = 0.7$
Upper and lower value of objective	$L_{C(x)}^T = 0.1555725$ $U_{C(x)}^T = 0.1561771$	$L_{C(x)}^T = 0.1503266$ $U_{C(x)}^T = 0.1509290$	$L_{C(x)}^T = 0.1480966$ $U_{C(x)}^T = 0.1486975$

Now using truth,indeterminacy and falsity membership function as mentioned in section 5 neutrosophic optimization problem can be formulated as similar as (10) and solving this optimal for different values of s, w, η, λ, design variables and objective functions can be obtained as follows.

Table 6. The optimum values of design variables for different values of w, η, λ and $s = 0.2$

Value of ε_i, ξ_i . Aspiration level, uncertainty level and desperation level $w_p = w_\sigma = w_\delta = w_\tau$ $= w_p = w_\sigma^1 = w_\delta^1 = w_\tau^1 = w$ $\eta_p = \eta_\sigma = \eta_\delta = \eta_\tau$ $= \eta_\sigma^1 = \eta_\delta^1 = \eta_\tau^1 = \eta$ $\lambda_p = \lambda_\sigma = \lambda_\delta = \lambda_\tau$ $= \lambda_\sigma^1 = \lambda_\delta^1 = \lambda_\tau^1 = \lambda$	$w = \eta = \lambda = 0.3$ $\varepsilon_i = \left(U_i^T - L_i^T\right) \times 0.1$ $\xi_i = \left(U_i^T - L_i^T\right) \times 0.1$	$w = \eta = \lambda = 0.5$ $\varepsilon_i = \left(U_i^T - L_i^T\right) \times 0.1$ $\xi_i = \left(U_i^T - L_i^T\right) \times 0.1$	$w = \eta = \lambda = 0.7$ $\varepsilon_i = \left(U_i^T - L_i^T\right) \times 0.1$ $\xi_i = \left(U_i^T - L_i^T\right) \times 0.1$
$x_1(in)$	0.3415895	0.3389869	0.3378618
$x_2(in)$	0.9535080	0.9463785	0.9433100
$x_3(in)$	2	2	2
$x_4(in)$	1.089426	1.080890	1.077210
$C(X)(\$)$	0.1420369	0.1388314	0.1374635

Where U_i *and* L_i are upper and lower bound of respective objective and constraints

Table 7. The optimum values of design variables for different values of w, η, λ and $s = 0.5$

Value of ε_i, ξ_i . Aspiration level uncertainty level and desperation level $w_p = w_\sigma = w_\delta = w_\tau$ $= w_p = w_\sigma^1 = w_\delta^1 = w_\tau^1 = w$ $\eta_p = \eta_\sigma = \eta_\delta = \eta_\tau$ $= \eta_\sigma^1 = \eta_\delta^1 = \eta_\tau^1 = \eta$ $\lambda_p = \lambda_\sigma = \lambda_\delta = \lambda_\tau$ $= \lambda_\sigma^1 = \lambda_\delta^1 = \lambda_\tau^1 = \lambda$	$w = \eta = \lambda = 0.3$ $\varepsilon_i = \left(U_i^T - L_i^T\right) \times 0.1$ $\xi_i = \left(U_i^T - L_i^T\right) \times 0.1$	$w = \eta = \lambda = 0.5$ $\varepsilon_i = \left(U_i^T - L_i^T\right) \times 0.1$ $\xi_i = \left(U_i^T - L_i^T\right) \times 0.1$	$w = \eta = \lambda = 0.7$ $\varepsilon_i = \left(U_i^T - L_i^T\right) \times 0.1$ $\xi_i = \left(U_i^T - L_i^T\right) \times 0.1$
$x_1(in)$	0.3422657	0.3396719	0.3385506
$x_2(in)$	0.9552806	0.9481638	0.9451009
$x_3(in)$	2	2	2
$x_4(in)$	1.091979	1.083465	1.079794
$C(X)(\$)$	0.1486395	0.1444629	0.1426829

Where U_i *and* L_i are upper and lower bound of respective objective and constraints.

Table 8. The optimum values of design variables for different values of w,η,λ and $s=0.8$

Value of ε_i,ξ_i. Aspiration level, uncertainty level and desperation level $w_p=w_\sigma=w_\delta=w_\tau$ $=w_p=w_\sigma^1=w_\delta^1=w_\tau^1=w$ $\eta_p=\eta_\sigma=\eta_\delta=\eta_\tau$ $=\eta_\sigma^1=\eta_\delta^1=\eta_\tau^1=\eta$ $\lambda_p=\lambda_\sigma=\lambda_\delta=\lambda_\tau$ $=\lambda_\sigma^1=\lambda_\delta^1=\lambda_\tau^1=\lambda$	$w=\eta=\lambda=0.3$ $\varepsilon_i=$ $\left(U_i^T-L_i^T\right)\times 0.1$ $\xi_i=$ $\left(U_i^T-L_i^T\right)\times 0.1$	$w=\eta=\lambda=0.5$ $\varepsilon_i=$ $\left(U_i^T-L_i^T\right)\times 0.1$ $\xi_i=$ $\left(U_i^T-L_i^T\right)\times 0.1$	$w=\eta=\lambda=0.7$ $\varepsilon_i=$ $\left(U_i^T-L_i^T\right)\times 0.1$ $\xi_i=$ $\left(U_i^T-L_i^T\right)\times 0.1$
$x_1(in)$	0.3429429	0.3403578	0.3392404
$x_2(in)$	0.9570581	0.9499542	0.9468969
$x_3(in)$	2	2	2
$x_4(in)$	1.094538	1.086046	1.082384
$C(X)(\$)$ 0.1556330	0.1503868	0.1503868	

Where U_i *and* L_i are upper and lower bound of respective objective and constraints

From the above results it is clear that whenever we chose $w=\eta=\lambda=0.7$ and $s=0.2$ the of cost welding is minimum most. Also it has been observed that cost of welding is decreased by higher value of aspiration level, uncertainty level and desperation level for a particular value of parameter ‘s’.

7. Conclusions

In this paper, we have proposed a method to solve welded beam design in fully neutrosophic environment. Here generalized neutrosophic number has been considered for deflection and stress parameter. The said model is solved by single objective parametric neutrosophic optimization technique and result is calculated for different parameter. The main advantage of the described method is that it allows us to overcome the actual limitations in a problem where impreciseness of supplied data are involved during the specification of the objectives. This approximation method can be applied to optimize different models in various fields of engineering and sciences.

Acknowledgement

The research work of Mridula Sarkar is financed by Rajiv Gandhi National Fellowship (F1-17.1/2013-14-SC-wes-42549/(SA-III/Website)), Govt of India.

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