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Realistic model using difference equations for repayment of loans in finance

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Abstract

The purpose of this paper is to find simple ways of solving problems in finance by the difference equations. As difference equations have discrete variables they are useful for developing computer program. The main contribution of this paper is the simplicity of the method presented which gives opportunity to create formulae for cases which are specialized and more realistic. In this article we formulate a mathematical model for repayment of loans by equal installments (such as EMI - Equated Monthly Installments, Equated Fortnightly Installments and Equated Quarterly Installments).

The aim of this article is to develop computer program which can be used easily. The software can be helpful to consumers prone to fuzzy math. The paper is concluded by giving a comparative study table to weigh the various paying options.

Key words: Amortization, EMI (equated monthly installments), fuzzy math.

1. Introduction

Finance problems are solved in various ways but here we have used difference equations methods to construct our mathematical models and then changed them according to our requirements. The people prone to fuzzy math in household finance are those who cannot understand mathematics properly, can get guidance from this paper.

Here the model[1-4] under consideration for repayment of loans is a method called amortization (repaying a debt – principals loaded with interest by a series of periodic equal installments). In this paper, we discuss and compare the various paying options such as monthly, fortnightly & quarterly installments so that we can estimate the cost of borrowing[5-7].

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2. *Preliminaries :*

2.1 *Basic Concepts of Difference Equations*

Definition:
Difference Equations:

An equation relating the value of the function and one or more of its differences

$\Delta y, \Delta^2 y, \ldots$ for each x-value of some set of numbers S(for which each of these function is defined) is called a difference equation over the set S.

Ex $\quad \Delta^2 y(x) + 2\Delta y(x) + y(x) = 0$

Linear First order Difference equation and its solution:

The linear first order difference equation has the form

$$f_0(k)\, y_{k+1} + f_1(k)\, y_k = g(k), \quad k=0,1,2,3,\ldots$$

Over the indicated set of k-values, where the function f_0 and f_1 are non zero functions.

Case 1:

f_0, f_1 and g are constant functions. Then the equation can be written in the form

$y_{k+1} = Ay_k + B$, where A & B are arbitrary constants

Solution is

$y_k = A^k y_0 + B(1-A^k)/(1-A)$, if $A \neq 1$, $k=0,1,2,..$

Or $\quad y_k = y_0 + B\,k \quad$ if $A=1$

Case 2: The most general form Is

$$y_{k+1} - a_k y_k = r_k, \quad k=0,1,2,3\ldots$$

Solution is

$y_k = u_k v_k$, where $u_0=c$, $u_k = c \prod_{j=0}^{k-1} a_j$ also $y_0=c$

& $\quad v_k = (\Delta^{-1} r_k)/u_{k+1} + c$

Linear Differene Equaion of Second Order:

Here we consider the second order linear difference equation with constant coefficients:

$$y_{k+2} + a_1 y_{k+1} + a_2 y_k = r_k$$

The solution is given by

$$y_k = c_1 y^{(1)} + c_2 y^{(2)} + y^*$$

Where y^* is a particular solution and $y^{(1)}$ and $y^{(2)}$ form a fundamental set of solutions (solution of corresponding homogeneous equation).

3. *Main Model :*

Definition:
3.1 *Amortization:*

Amortization is a method of repaying a debt, including both principal and interest by a series of periodic payments, usually equal in amount, each of which is part payment to reduce outstanding principal.

The EMI (Equated Monthly Installment) is one of its examples.

3.2 *Construction of an equivalent mathematical model :*

To construct the model let A be the loan (debt) to be repaid, interest charges be at compound rate (i)per conversion period, and the periodic payment (made at the end of each conversion period) be R. Let P_n be the outstanding principal after the nth payment of R (or equivalently at the beginning of the (n+1)th period). Before the payment is made, the debt increases by the interest due on the principal P_n. After the (n+1)th payment of R the outstanding debt is P_{n+1}. Therefore we have

$$P_{n+1} = P_n + (i)P_n - R$$

Or $$P_{n+1} = (i+1)P_n - R$$

With $$P_0 = A$$

This is a first order difference equation with constant coefficients. Therefore, its solution is given by

$$P_n = A(1+i)^n + (-R)/(1-1-i) * (1-(1+i)^n)$$

Or $$P_n = A(1+i)^n - R/(i)* ((1+i)^n -1)$$

This is the required solution. The first term on the right is the amount to which the initial debt accumulate (at the compound interest rate(i)) after n periods, the second term is the amount to which the n periodic payments accumulate in this same time. Their difference is the same remaining debt.

If we want to amortize the periodic payment R in order to amortize the debt A by exactly n payments then we have

$$P_n = 0,$$

Therefore from above equation we have

$$0 = A(1+i)^n - R/(i)* ((1+i)^n-1)$$

Or $$R=Ai(1+i)^n/((1+i)^n-1) \quad (3.2.1)$$

$$R = Ai/(1-(1+i)^{-n})$$

Or $$R = A/(amor)$$

Where $(amor) = (1-(1+i)^{-n})/i$

The number (amor) is referred to as the amortization factor and it is present value of an annuity of 1 per period for n periods at rate (i).

The above formula (3.2.1) can be specialized according to our requirements.

3.3 *Formula For Equal Monthly Installment:*

We replace i by i / (12x100)

& n by 12n in formula (3.2.1) to get EMI formula

$$R = A(i/1200)(1+i/1200)^{12n}/((1+i/1200)^{12n}-1) \quad (3.3.1)$$

where n is the number of installments by which we require to repay the debt.

Similarly, by changing i & n, we get various option for fortnightly and quarterly payment.

3.4 *Development of Software :*

The above mathematical model of finding the EMI has been converted into the executable computer software using C++ computer language. The software is user friendly and has been developed with various options such as one program for finding EMI only whereas one other program giving EMIs in the form of chart for various installments of 6,12,18 & 24 etc. Software has also option for repayment of debt in fortnightly installments and in quarterly installments. Even the fortnightly and quarterly installments programs are in chart form. The algorithm of main program of EMI is given below:

3.4.1 Algorithm:

(1) To calculate EMI (equated monthly installments) we require the following data
  (i) Loan Amount
  (ii) rate of interest per annum
  (iii) the number of installments in which the loan is to be repaid.
(2) Loan amount = principal
  Rate of interest =rate
  Number of years=n
(3) ratio =(rate/100)/12
  Term = $(1+\text{ratio})^{12n}$
(4) number of installments = 12*n
  Installment = principal *ratio *term/(term-1)
  Print EMI= installment

4. Discussion

4.1 *Importance of this study :*

Let us take an example to show how this study can be helpful by making a person vigilant while paying his loan. Suppose a person takes a loan of Rs.1,00,000/- from some finance company and the company says we are not charging you anything extra only interest of 12% per annum and your 6 monthly installments will be Rs.17667. But actually it is duping the client by not calculating on reducing balance.

By amortization, one can easily find the monthly installment by using EMI formula (3.3.1) for the above example as under -

Here A= 100,000, loan to be repaid in 6 monthly installments at the rate of 12% per annum then we have to take (i) =12 & n=6 in EMI formula (3.3.1) & get

R = (100000) /(amor)
Where
$(\text{amor})=(1-(1+(12/(12*100))^{12n})/(12/(12*100))$
R = 17255

Thus we see that we are asked to pay installment of Rs.17667 instead of Rs.17255.

4.2 *Comparative Tables For Various Options:*

Loan Amount Rs100,000/-
Rate of Interest 10% per annum

4.2.1 For Monthly Installments :

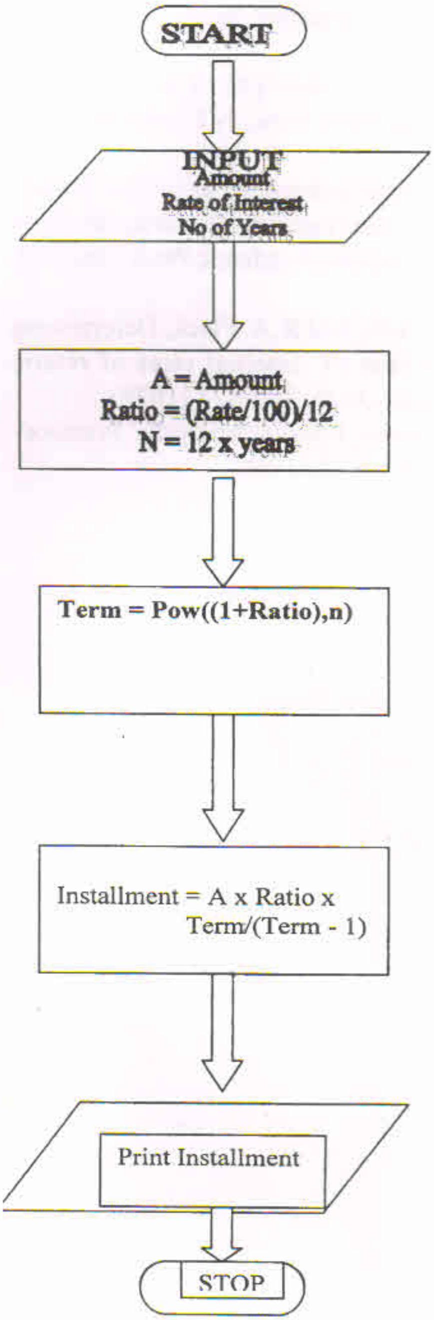
No. of Installments	Installment Amount (Rs)	Total Amount Paid (Rs)
6	17156.14	102936.84
12	8791.59	105499.08
18	6005.71	108102.78
24	4614.48	110747.52

4.2.2 For Fortnightly Installments :

No. of Installments	Installment Amount (Rs)	Total Amount Paid (Rs)
6	16910.56	101463.36
12	8560.74	102728.88
18	5778.05	104004.90
24	4387.14	105291.36

APPENDIX

Flowchart:



4.2.3 For Quarterly Installments :

No. of Installments	Installment Amount (Rs)	Total Amount Paid (Rs)
6	18155	108930
12	9748.71	116984.52
18	6967.01	125406.18
24	5591.28	134190.72

From the above tables we can weigh which paying option suits us best so as to minimize our cost of borrowing. People have misconception and they tend to underestimate the cost of short borrowings irrespective of their paying capacity. However, from the above data it is very evident that repayment period should be as short as per repaying capacity. The data bear out that short term debt is more opportunistic.

5. Conclusion

The aim of this paper was to make actual working model which can be used to solve some finance problems of repaying the loans and develop user friendly executable computer software.

The computer program shows different ways of repaying the loan amount. In fact it can be used by the common man to find EMI. Based on the model the above tables can be used to analyze and select the paying option as per the requirement. With the advent of loan culture, as it is easy to get loans now days, every one is taking loans in some way or the other and this paper can be of immense use to decide the most suitable way to repay in installments. Hence in present day scenario it is useful to the society at large, especially people prone to fuzzy math.

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7. References

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