Ultra Scientist Vol. 26(1)A, 35-38 (2014).

Some new Bilateral Generating Relations Involving Hypergeometric Function and H-Function of one and two Variables

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(Acceptance Date 8th January, 2014)

Abstract

In this paper, we establish some new bilateral generating relations involving additional complete hypergeometric functions of three variables and H-function of one variable.

1 Introduction

$$(a,n) = (a)(a+1)(a+2),\dots,(a+n-1);\ (a,0) = 1$$

Where a is arbitrary and n is positive integer, then the additional functions H_A, H_B *and* H_C of three variables have been defined by Shrivastava (3,1967a,p. 17-32), and Shrivastava and Manocha (2, p. 68,69) are respectively as follows;

$$H_A[\alpha,\beta,\beta';\gamma,\gamma';x,y,z] = \sum_{m,n,p=0}^{\infty} \frac{(\alpha,m+p)(\beta,m+n)(\beta',n+p)}{(\gamma,m)(\gamma',n+p)(1,m)(1,n)(1,p)}\, x^m y^n z^p, \quad (1.1)$$

$$H_B[\alpha,\beta,\beta';\gamma_1,\gamma_2,\gamma_3;x,y,z] = \sum_{m,n,p=0}^{\infty} \frac{(\alpha,m+p)(\beta,m+n)(\beta',n+p)}{(\gamma_1,m)(\gamma_2,n)(\gamma_3,p)(1,m)(1,n)(1,p)}\, x^m y^n z^p, \quad (1.2)$$

$$H_C[\alpha,\beta,\beta';\gamma;x,y,z] = \sum_{m,n,p=0}^{\infty} \frac{(\alpha,m+p)(\beta,m+n)(\beta',n+p)}{(\gamma,m+n+p)(1,m)(1,n)(1,p)}\, x^m y^n z^p, \quad (1.3)$$

With all other conditions already detailed by Shrivastava and Manocha[2], Fox (1, p. 408) has defined following the H-function of one variable

$$H[u] = H_{p,q}^{m,n}\left[u\ \middle|\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right] = \frac{1}{2\pi\omega}\int_L \theta(s) x^s\, ds, \quad (1.4)$$

Where,

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)},$$

Along with the conditions given in Fox [1]

2. *Formula Used:*

The following formula are required in the present investigation Shrivastava and Manocha (2,p.22.34),

$$\Gamma(\alpha,-n) = (\alpha,-n)\Gamma(\alpha), \quad (2.1)$$

$$\sum_{n=0}^{\infty}\frac{(\propto)_n t^n}{n!}=(1-t)^{-\propto}, \quad (2.2)$$

$$\sum_{n=0}^{\infty}\frac{(\propto)_n(-t)^n}{n!}=(1+t)^{-\propto}, \quad (2.3)$$

$$(\alpha,-n)=\frac{(-1)^n}{(1-\alpha,n)}, \quad (2.4)$$

$$(\gamma,-k)(\gamma-k,m)=(\gamma,m-k)=\frac{(-1)^k(\gamma,m)}{(1-\gamma-m,k)}, \quad (2.5)$$

3. The following bilateral generating relations involving Additional complete hypergeometric functions of three variables and H-functions of one variable have been derived.

$$\sum_{r=0}^{\infty}\left(\frac{\omega t}{1.r}\right)^r H_A\,[\alpha,\beta,\beta';\gamma-r,\gamma';x,y,z]$$

$$\times H_{P+1,Q}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P,}(\gamma-r,o)\\(b_j,\beta_j)_{1,Q}\end{matrix}\right.\right]$$

$$=(1-\omega t)^{\gamma-1}H_A[\alpha,\beta,\beta';\gamma,\gamma';x(1+\omega t),y,z]$$

$$\times H_{P+1,Q}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P,}(\gamma,o)\\(b_j,\beta_j)_{1Q}\end{matrix}\right.\right], \quad (3.1)$$

$$\sum_{r=0}^{\infty}\left(\frac{-\omega t}{1.r}\right)^r H_A\,[\alpha,\beta,\beta';\gamma,\gamma'-r;x,y,z]$$

$$\times H_{P+1,Q}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P,}(\gamma-r,o)\\(b_j,\beta_j)_{1,Q}\end{matrix}\right.\right]$$

$$=(1-\omega t)^{\gamma-1}H_A[\alpha,\beta,\beta';\gamma,\gamma';x,y(1-\omega t),z(1-\omega t)]$$

$$\times H_{P+1,Q}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P,}(\gamma',o)\\(b_j,\beta_j)_{1,Q}\end{matrix}\right.\right], \quad (3.2)$$

$$\sum_{r=0}^{\infty}\left(\frac{\omega t}{1,r}\right)^r H_B\,[\alpha,\beta,\beta';\gamma_1-r,\gamma_2,\gamma_3;x,y,z]$$

$$\times H_{P,Q+1}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q},(1-\gamma_1+r,0)\end{matrix}\right.\right]$$

$$=(1+\omega t)^{\gamma_1-1}H_B[\alpha,\beta,\beta';\gamma_{11},\gamma_2,\gamma_3;x(1+\omega t),y,z]$$

$$\times H_{P,Q+1}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q},(1-\gamma_1,0)\end{matrix}\right.\right], \quad (3.3)$$

$$\sum_{r=0}^{\infty}\left(\frac{-\omega t}{1,r}\right)^r H_B\,[\alpha,\beta,\beta';\gamma_1\gamma_2-r,\gamma_3,;x,y,z]$$

$$\times H_{P,Q+1}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q},(1-\gamma_2+r,0)\end{matrix}\right.\right]$$

$$=(1-\omega t)^{\gamma_2-1}H_B[\alpha,\beta,\beta';\gamma_1,\gamma_2,\gamma_3;x,y(1-\omega t),z]$$

$$\times H_{P,Q+1}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q},(1-\gamma_2,0)\end{matrix}\right.\right], \quad (3.4)$$

$$\sum_{r=0}^{\infty}\left(\frac{\omega t}{1,r}\right)^r H_B\,[\alpha,\beta,\beta';\gamma_1\gamma_2,\gamma_3-r,x,y,z]$$

$$\times H_{P+1,Q}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P}(\gamma_3-r,0)\\(b_j,\beta_j)_{1,Q},\end{matrix}\right.\right]$$

$$=(1+\omega t)^{\gamma_1-1}H_B[\alpha,\beta,\beta';\gamma_1,\gamma_2,\gamma_3;x,y(1+\omega t)]$$

$$\times H_{P+1,Q}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P},(\gamma_3,0)\\(b_j,\beta_j)_{1,Q},\end{matrix}\right.\right], \quad (3.5)$$

$$\sum_{r=0}^{\infty}\left(\frac{-\omega t}{1,r}\right)^r H_C\,[\alpha,\beta,\beta';\gamma-r;x,y,z]$$

$$\times H_{P,Q+1}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1.Q},(1-\gamma+r,0)\end{matrix}\right.\right]$$

$$=(1+\omega t)^{\nu_1-1}H_C[\alpha,\beta,\beta';\gamma,$$
$$x(1-\omega t),y(1-wt),z(1-wt)]$$

$$\times H_{P,Q+1}^{M,N}\left[u\left|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q},(1-\gamma,0)\end{matrix}\right.\right],$$

Proof :

Proof of (3.1), considering

$$\Delta= \sum_{r=0}^{\infty} \left(\frac{\omega t}{1,r}\right)^{r} H_A\,[\alpha, \beta, \beta'; \gamma - r, \gamma'; x, y, z]$$

$$\times H_{P+1,Q}^{M,N}\left[u \middle| \begin{matrix} (a_j, \alpha_j)_{1,P} (\gamma - r, 0) \\ (b_j, \beta_j)_{1,Q} \end{matrix}\right]$$

Expressing H_A in series from on using (1.1) and H-function on using (1.4) and using (2.1), we get

$$\Delta = \sum_{r=0}^{\infty} \left(\frac{\omega t}{1, r}\right)^{r} \sum_{m,n,p=0}^{\infty} \frac{(\alpha, m+p)\,(\beta, m+n)\,(\beta', n+p)}{(\gamma - r, m)\,(\gamma', n+p)(1, m)(1, n)(1, p)} x^m y^n z^p$$
$$\times \left[\frac{1}{2\pi i} \int_L \theta(s) \left\{\frac{1}{(\gamma - r)\Gamma(\gamma)}\right\} u^s ds\right],$$

Now interchange the order of summation and integration and using (2.5), we get

$$\Delta= \frac{1}{2\pi i} \int_L \theta(s) \frac{1}{\Gamma(\gamma)} u^s \times \sum_{m,n,p=0}^{\infty} \frac{(\alpha, m+p)\,(\beta, m+n)\,(\beta', n+p)}{(\gamma, m)\,(\gamma', n+p)(1, m)(1, n)(1, p)} x^m y^n z^p$$
$$\times \left[\sum_{r=0}^{\infty} \frac{(1-\gamma-m, r)(-\omega t)^r t}{(1,r)}\right] ds,$$

Again applying (2.2), we find that,

$$\Delta= \frac{1}{2\pi i} \int_L \theta(s) \frac{1}{\Gamma(\gamma)} u^s \times \sum_{m,n,p=0}^{\infty} \frac{(\alpha, m+p)\,(\beta, m+n)\,(\beta', n+p)}{(\gamma, m)\,(\gamma', n+p)(1, m)(1, n)(1, p)} x^m y^n z^p$$
$$\times \left[(1 + \omega t)^{-(1-\gamma-m)}\right] ds,$$

Which, in the light of (1.1) and (1.4), provides (3.1). Proceeding on similar lines, results (3.2) to (3.6) can be derived with the help of the formulae given in section 1 and 2.

References

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