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Some new Linear Generating Relations Involving H-Function of one Variables

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Abstract

The object of this paper is to evaluate some new generating relations involving H-function of one variable some special cases have also been derived.

1. Introduction

Fox[1] in (1961) has defined following the H-function of one variable

$$H_{p,q}^{m,n}\left[x\ \left|\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right.\right]=\frac{1}{2\pi\omega}\int_L\theta(s)x^s\,ds, \quad (1.1)$$

Where,

$$\theta(s)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)},$$

2. Formula Used:

The following formula are required in the present investigation

$$\sum_{n=0}^{\infty}\frac{(\propto)_n t^n}{n!}=(1-t)^{-\propto}, \quad (2.1)$$

$$(\propto)_n=\frac{\Gamma\alpha+n}{\Gamma\alpha}, \quad (2.2)$$

$$2F_1\left[\begin{matrix}-n,a;\\1+a+n;\end{matrix}-1\right]=\frac{(1+a)_n}{\left(1+\frac{a}{2}\right)_n}, \quad (2.3)$$

$$1F_0[a;-;z]=(1-z)^{-a}, \quad (2.4)$$

$$0F_0[-;-;z]=e^z, \quad (2.5)$$

$$\mathrm{H}_{0,1}^{1,0}\left[x\left|\begin{matrix}-\\(b,\beta\end{matrix}\right.\right]=\beta^{-1}x^{b/\beta}\ exp\left(-x^{\frac{1}{\beta}}\right), \quad (2.6)$$

$$\mathrm{H}_{1,1}^{1,1}\left[x\left|\begin{matrix}1-a,1\\(0,1)\end{matrix}\right.\right]=\Gamma a(1+x)^{-a}$$
$$=\Gamma a\ 1F_0[a;\ -;-x], \quad (2.7)$$

3. Generating Relations: In this section, we establish the following generating relations involving H-function of one variable:[2-4]

$$\sum_{n=0}^{\infty}\left(\frac{ut}{v}\right)^n\frac{1}{n!}2F_1\left[\begin{matrix}-n,a;\\1+a+n;\end{matrix}-1\right]H_{P+1,Q}^{M,N+1}\left[x\left|\begin{matrix}\left(\frac{-a}{2}-n,0\right),(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{matrix}\right.\right]$$

$$=\left(\frac{v}{v-ut}\right)^{1+a}H_{P+1,Q}^{M,N+1}\left[x\left|\begin{matrix}\left(\frac{-a}{2},0\right)(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{matrix}\right.\right] \quad (3.1)$$

$|\arg(x)| < \frac{1}{2}\pi M;$

$$\sum_{\iota=0}^{\infty}\left(\frac{ut}{v}\right)^{n}\frac{1}{n!}2F_1\left[{-n,a; \atop 1+a+n;}-1\right]H_{P,Q+1}^{M+1,N}\left[x\left|{(a_j,\alpha_j)_{1,P} \atop \left(1+\frac{a}{2}+n,0\right),(b_j,\beta_j)_{1,Q}}\right.\right]$$

$$=\left(\frac{v}{v-ut}\right)^{1+a}H_{P,Q+1}^{M+1,N}\left[x\left|{(a_j,\alpha_j)_{1,P} \atop \left(1+\frac{a}{2},0\right),(b_j,\beta_j)_{1,Q}}\right.\right] \tag{3.2}$$

$|\arg(x)| < \frac{1}{2}\pi M;$

$$\sum_{n=0}^{\infty}\left(\frac{ut}{v}\right)^{n}\frac{1}{n!}2F_1\left[{n,-a; \atop 1-a-n;}-1\right]H_{P,Q+1}^{M+1,N}\left[x\left|{(a_j,\alpha_j)_{1,P} \atop (a+n,0),(b_j,\beta_j)_{1,Q}}\right.\right]$$

$$=\left(\frac{v}{v-ut}\right)^{\frac{a}{2}}H_{P,Q+1}^{M+1,N}\left[x\left|{(a_j,\alpha_j)_{1,P} \atop (a,0),(b_j,\beta_j)_{1,Q}}\right.\right] \tag{3.3}$$

$|\arg(x)| < \frac{1}{2}\pi M;$

$$\sum_{n=0}^{\infty}\left(\frac{ut}{v}\right)^{n}\frac{1}{n!}2F_1\left[{n,-a; \atop 1-a-n;}-1\right]H_{P+1,Q}^{M,N+1}\left[x\left|{(1-a-n),(a_j,\alpha_j)_{1,P} \atop (b_j,\beta_j)_{1,Q}}\right.\right]$$

$$=\left(\frac{v}{v-ut}\right)^{\frac{a}{2}}H_{P+1,Q}^{M,N+1}\left[x\left|{(1-a,0),(a_j,\alpha_j)_{1,P} \atop (b_j,\beta_j)_{1,Q}}\right.\right], \tag{3.4}$$

Proof (3.1):

Let L.H.S. of (3.1)

$$\sum_{n=0}^{\infty}\left(\frac{ut}{v}\right)^{n}\frac{1}{n!}2F_1\left[{-n,a; \atop 1+a+n;}-1\right]H_{P+1,Q}^{M,N+1}\left[x\left|{\left(\frac{-a}{2}-n,0\right),(a_j,\alpha_j)_{1,P} \atop (b_j,\beta_j)_{1,Q}}\right.\right]$$

Now, replace H-function by its contour integration and using (2.3), (2.2), (2.1) and interchange the order of summation and integration, we get

$$= \frac{1}{2\pi i}\int_L \theta(s)x^s\,\Gamma\left(1+\frac{a}{2}\right)\left[\sum_{n=0}^{\infty}\left(\frac{ut}{v}\right)^n\frac{1}{n!}(1+a)_n\right]ds$$

$$= \left(\frac{v}{v-ut}\right)^{1+a}\frac{1}{2\pi i}\int_L \theta(s)x^s\,\Gamma\left(1+\frac{a}{2}\right)ds$$

$$= \left(\frac{v}{v-ut}\right)^{1+a} H^{M,N+1}_{P+1,Q}\left[x\left|\begin{matrix}\left(\frac{-a}{2},0\right)(a_j,\alpha_j)_{1,P}\\ (b_j,\beta_j)_{1,Q}\end{matrix}\right.\right]$$

R.H.S. of (3.1) Proceeding on similar lines, we can establish result (3.2), (3.3), (3.4)

4.Special Cases:

$M=N=P=Q=1,\ \alpha_j=\beta_j=1, a_j=1-c, b_j=0, u=v=1$ in equation (3.1) we get following generating relation.

$$\sum_{n=0}^{\infty}(t)^n\frac{1}{n!}2F_1\left[\begin{matrix}-n,a;\\1+a+n;\end{matrix}-1\right]H^{1,2}_{2,1}\left[x\left|\begin{matrix}\left(\frac{-a}{2}-n,0\right),(1-c,1)\\(0,1)\end{matrix}\right.\right]$$

$$=\sum_{n=0}^{\infty}(t)^n\frac{1}{n!}\frac{(1+a)_n}{\left(1+\frac{a}{2}\right)_n}\Gamma\left(1+\frac{a}{2}+n\right)H^{1,1}_{1,1}\left[x\left|\begin{matrix}(1-c,1)\\(0,1)\end{matrix}\right.\right]$$

$$=(1-t)^{-(a+1)}\frac{1}{2}\Gamma\left(\frac{a}{2}\right)\Gamma(\mathrm{c})(1+x)^{-c}$$

$$=\frac{a}{2}\Gamma\left(\frac{\mathrm{a}}{2}\right)\Gamma(\mathrm{c})1\mathrm{F}_0[(\mathrm{a}+1);-;\mathrm{t}]1\mathrm{F}_0[(\mathrm{c};-;\mathrm{x}], \tag{4.1}$$

In equation (3.2) put $M=Q=1, N=P=0, b_j=b, \beta_j=\lambda, u=v=1$ and using (2.3), (2.4), (2.5), (2.6) we get following generating relation.

$$\sum_{n=0}^{\infty}(t)^n\frac{1}{n!}2F_1\left[\begin{matrix}-n,a;\\1+a+n;\end{matrix}-1\right]H^{2,0}_{0,2}\left[x\left|\begin{matrix}-\\\left(1+\frac{a}{2}+n,0\right),(b,\lambda)\end{matrix}\right.\right]$$

$$= \sum_{n=0}^{\infty}(t)^n \frac{1}{n!}\frac{(1+a)_n}{\left(1+\frac{a}{2}\right)_n}\Gamma\left(1+\frac{a}{2}+n\right)H_{0,1}^{1,0}\left[x\,\middle|\,\overline{(b,\lambda)}\right]$$

$$= (1-t)^{-(a+1)}\Gamma(1+\ a/2)\lambda^{-1}\ \ x^{\frac{b}{\lambda}} exp\left(-x^{\frac{1}{\lambda}}\right)$$

$$= \frac{a}{2}\Gamma\frac{a}{2}\lambda^{-1}x^{\frac{b}{\lambda}}1F_0[a+1;-;t]0F_0\left[-;-;-x^{\frac{1}{\lambda}}\right], \tag{4.2}$$

In equation (3.3) put $M = Q = 1, N = P = 0, b_j = b, \beta_j = \lambda, u = v = 1$ and using (2.3), (2.4), (2.5), (2.6) we get following generating relation

$$\sum_{n=0}^{\infty}(t)^n\frac{1}{n!}2F_1\left[\begin{matrix}n,-a;\\1-a-n;\end{matrix}-1\right]H_{0,2}^{2,0}\left[x\,\middle|\,\overline{(a+n,0),(b,\lambda)}\right]$$

$$= (1-t)^{-\frac{a}{2}}\Gamma(a)\lambda^{-1}\ \ x^{\frac{b}{\lambda}}\ exp\left(-x^{\frac{1}{\lambda}}\right)$$

$$= \Gamma(a)\lambda^{-1}x^{\frac{b}{\lambda}}1F_0\left[\frac{a}{2};-;t\right]0F_0\left[-;-;-x^{\frac{1}{\lambda}}\right], \tag{4.3}$$

In equation (3.4) put $M= N = P = Q = 1, a_j = 1-c, b_j = 0, \alpha_j = \beta_j = 1, u = v = 1$ and using section 2 we get following condition.

$$\sum_{n=0}^{\infty}(t)^n\frac{1}{n!}2F_1\left[\begin{matrix}n,-a;\\1-a-n;\end{matrix}-1\right]H_{2,1}^{1,2}\left[x\,\middle|\,\begin{matrix}(1-a-n,0)(1-c,1)\\(0,1)\end{matrix}\right]$$

$$= (1-t)^{-\frac{a}{2}}\Gamma(a)\Gamma(c)\ \ (1+x)^{-c}$$

$$= \Gamma(a)\Gamma(c)\ 1F_0\left[\frac{a}{2};-;t\right]1F_0[c;-;-x], \tag{4.4}$$

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